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$F(\mathcal{R})$ Supergravity and Early Universe: the Meeting Point of Cosmology and High-Energy Physics

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Abstract

Cosmological inflation in supergravity is a great window to High-Energy Physics beyond the Standard Model. I review a construction of the new $F(\mathcal{R})$ supergravity theories, and consider some of their applications to cosmology. The $F(\mathcal{R})$ supergravity theories are the $N = 1$ locally supersymmetric extensions of the well studied $f(R)$ gravity theories in four space-time dimensions, which are often used for 'explaining' inflation and Dark Energy. A manifeslty supersymmetric description of $F(\mathcal{R})$ supergravities exists in terms of $N = 1$ superfields, by using the (old) minimal Poincaré supergravity in curved superspace. We find that any $F(\mathcal{R})$ supergravity is classically equivalent to the Poincaré-type matter-coupled $N = 1$ supergravity via the superfield Legendre-Weyl-Kähler transformation. The (non-trivial) Kähler potential and the scalar superpotential of the inflaton superfield are determined in terms of the original holomorphic $F(\mathcal{R})$ function. The conditions for stability, the absence of ghosts and tachyons are also found. No-scale $F(\mathcal{R})$ supergravity is constructed too. Three different examples of $F(\mathcal{R})$ supergravity theories are studied in detail. The first example is devoted to recovery of the standard (pure) $N = 1$ supergravity with a negative cosmological constant from $F(\mathcal{R})$ supergravity. As the second example, a generic $\mathcal{R}^2$ supergravity is investigated, and the existence of the AdS bound on the scalar curvature is discovered. As the third example, a simple viable realization of chaotic inflation in supergravity is given, via an embedding of the Starobinsky inflationary model into the $F(\mathcal{R})$ supergravity. It is also found that $F(\mathcal{R})$ supergravity can have a positive cosmological constant in the low-curvature regime. Our approach does not introduce new exotic fields or new interactions, beyond those already present in (super)gravity. A nonminimal scalar-curvature coupling in gravity and supergravity, Higgs inflation and its correspondence to Starobinsky inflation in $f(R)$ gravity and $F(\mathcal{R})$ supergravity, respectively, are establlished. In the outlook, reheating and particle production in the post-inflationary universe, CP-violation, origin of baryonic asymmetry, lepto- and baryo-genesis are briefly discussed. $F(\mathcal{R})$ supergravity shows promise for possible solutions to those outstanding problems.

CONTENTS

This review is based on the original papers [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11] and the invited seminar talks given by the author at the CERN Theory Division in Geneva (Switzerland), the DESY Theory Group in Hamburg, the Max-Planck Institutes for Physics and Gravitational Physics in Munich and Golm, respectively, Heidelberg University (Germany), The University of Tokyo, Waseda University, Tokyo Institute of Technology, Tokyo University of Science, KEK Theory Group and Yukawa Institute of Theoretical Physics in Kyoto (Japan).

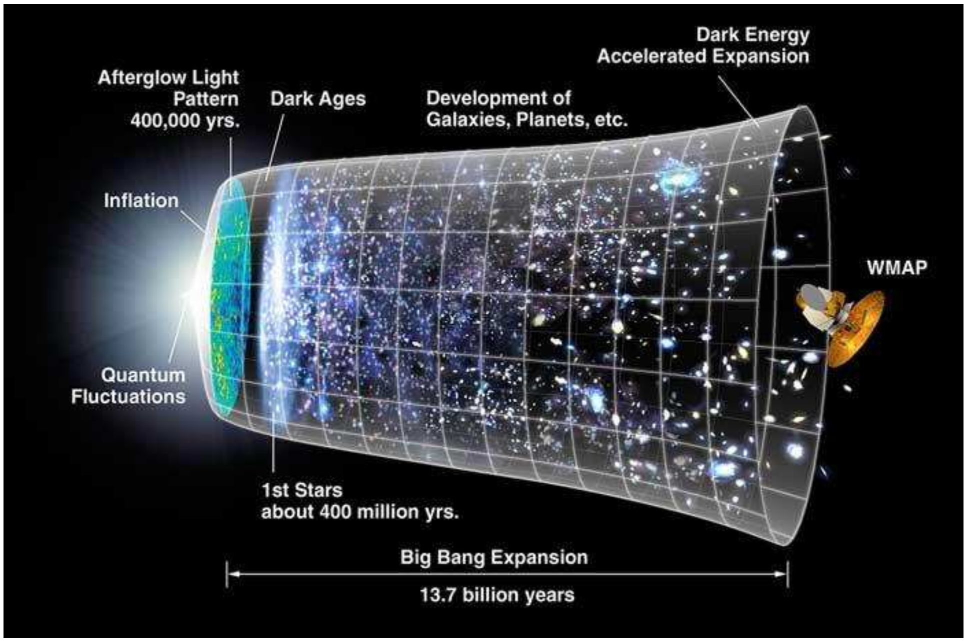

Figure 1: Timeline of the universe according to WMAP [12]

1 Introduction and Motivation

A very brief history of our universe is given by the standard Fig. 1 taken from the website of the Wilkinson Microwave Anisotropy Probe (WMAP) [12]. We focus on the field-theoretical description of the inflationary phase of early universe and its post-inflationary dynamics (reheating and particle production) in the context of supergravity. To begin with, let us first introduce some basics of inflation.

Cosmological inflation (a phase of ‘rapid’ quasi-exponential accelerated expansion of universe) [13, 14, 15] predicts homogeneity of our universe at large scales, its spatial flatness, large size and entropy, and the almost scale-invariant spectrum of cosmological perturbations, in good agreement with the WMAP measurements of the CMB radiation spectrum [12, 16]. Inflation is also the only known way to generate structure formation in the universe via amplifying quantum fluctuations in vacuum.

However, inflation is just the cosomological paradigm, not a theory! The known field-theoretical mechanisms of inflation use a slow-roll scalar field ϕ (called *inflaton*) with proper scalar potential $V(\phi)$ [14, 15].

The scale of inflation is well beyond the electro-weak scale, ie. is well beyond the Standard Model of Elementary Particles! Thus the inflationary stage in the early universe is the most powerful High-Energy Physics (HEP) accelerator in Nature (up to $10^{10}\ TeV$). Therefore, inflation is the great and unique window to very HEP!

The nature of inflaton and the origin of its scalar potential are the big mysteries.

Throughout the paper the units $\hbar = c = 1$ and the spacetime signature $(+,-,-,-)$

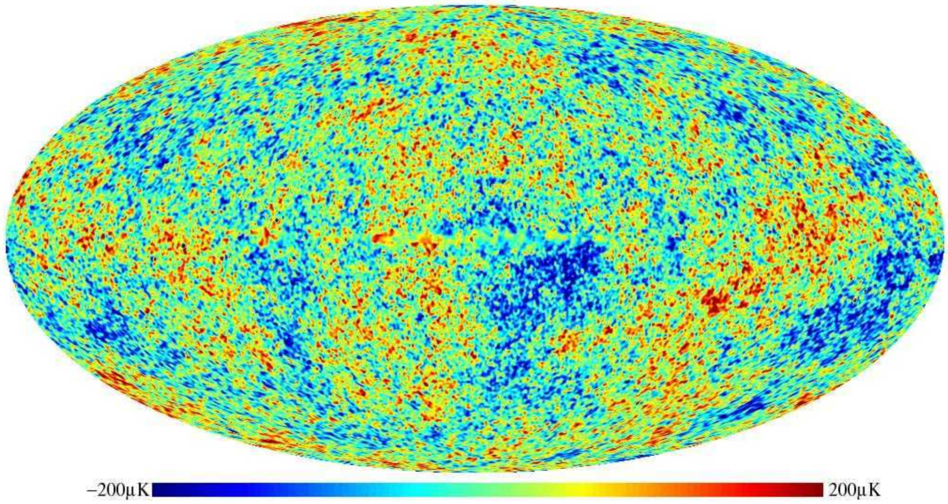

Figure 2: CMB from WMAP [12]: Mona Lisa of Cosmology

are used. See ref. [17] for our use of Riemann geometry of a curved spacetime.

The Cosmic Microwave Background (CMB) radiation picture from the WMAP (Fig. 2) is one of the main sources of data about early universe. Deciphering this picture in terms of density perturbations, gravity wave polarization, power spectrum and its various indices is a formidable task. It also requires the heavy CMB mathematical formalism based on General Relativity — see eg., the textbooks [18, 19, 20]. Fortunately, we do not need that formalism for our purposes, since the relevant indices can also be introduced in terms of the inflaton scalar potential (Sec. 4). We assume that inflation did happen. There exist many inflationary models — see eg. the textbook [15] for their description and comparison (without supersymmetry). Our aim is a viable theoretical description of inflation in the context of supergravity.

The main Cosmological Principle of a spatially homogeneous and isotropic $(1+3)$-dimensional universe (at large scales) gives rise to the FLRW metric

$$ds^2_{\rm FLRW} = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \tag{1}$$

where the function $a(t)$ is known as the scale factor in 'cosmic' (comoving) coordinates (t, r, θ, ϕ), and k is the FLRW topology index, $k = (-1, 0, +1)$. The FLRW metric (1) admits the six-dimensional isometry group G that is either $SO(1,3)$, $E(3)$ or $SO(4)$, acting on the orbits $G/SO(3)$, with the spatial three-dimensional sections H^3, E^3 or S^3, respectively. The Weyl tensor of any FLRW metric vanishes,

$$C^{\rm FLRW}_{\mu\nu\lambda\rho} = 0 \tag{2}$$

where $\mu, \nu, \lambda, \rho = 0, 1, 2, 3$. The early universe inflation (acceleration) means

$$\ddot{a}(t) > 0 \ , \ \text{or equivalently} \ , \ \frac{d}{dt}\left(\frac{H^{-1}}{a}\right) < 0 \tag{3}$$

where $H = \dot{a}/a$ is called Hubble function. We take $k = 0$ for simplicity. The amount of inflation (called the *e-foldings* number) is given by

$$N_e = \ln \frac{a(t_{\rm end})}{a(t_{\rm start})} = \int_{t_{\rm start}}^{t_{\rm end}} H \, dt \approx \frac{1}{M_{\rm Pl}^2} \int_{\phi_{\rm end}}^{\phi} \frac{V}{V'} \, d\phi \tag{4}$$

Next, a few words about our strategy. It is well recognized by now that one has to go beyond the Einstein-Hilbert action for gravity, both from the experimental viewpoint (eg.,because of Dark Energy) and from the theoretical viewpoint (eg., because of the UV incompleteness of quantized Einstein gravity, and the need of its unification with the Standard Model of Elementary Particles).

In our approach, the origin of inflation is purely *geometrical*, ie. is closely related to space-time and gravity. It can be technically accomplished by taking into account the higher-order curvature terms on the left-hand-side of Einstein equations, and extending gravity to supergravity. The higher-order curvature terms are supposed to appear in the gravitational effective action of Quantum Gravity. Their derivation from Superstring Theory may be possible too. The true problem is a *selection* of those high-order curvature terms that are physically relevant or derived from a fundamental theory of Quantum Gravity.

There are many phenomenological models of inflation in the literature, which usually employ some new fields and new interactions. It is, therefore, quite reasonable and meaningful to search for the *minimal* inflationary model building, by getting most economical and viable inflationary scenario. I am going to use the one proposed the long time ago by Starobinsky [21, 22], which does not use new fields (beyond a spacetime metric) and exploits only gravitational interactions. I also assume that the general coordinate invariance in spacetime is fundamental, and it should not be sacrificed. Moreover, it should be extended to the more fundamental, *local* supersymmetry that is known to imply the general coordinate invariance. It thus leads us to supergravity which, in addition, automatically has several viable candidates for *Dark Matter* particle (see Sec. 15 for more).

On the theoretical side, the available inflationary models may be also evaluated with respect to their *"cost"*, ie. against what one gets from a given model in relation to what one puts in! Our approach does *not* introduce new fields, beyond those already present in gravity and supergravity. We also exploit (super)gravity interactions *only*, ie. do *not* introduce new interactions, in order to describe inflation.

Before going into details, let me address two common prejudices and objections.

The higher-order curvature terms are usually expected to be relevant near the spacetime curvature singularities. It is also quite possible that some higher-derivative gravity, subject to suitable constraints, could be the effective action to a quantized theory

of gravity, [1] like eg., in String Theory. However, there are also some common doubts against the higher-derivative terms, in principle.

First, it is often argued that all higher-derivative field theories, including the higher-derivative gravity theories, have ghosts (i.e. are unphysical), because of Ostrogradski theorem (1850) in Classical Mechanics. As a matter of fact, though the presence of ghosts is a generic feature of the higher-derivative theories indeed, it is not always the case, while many explicit examples are known (Lovelock gravity, Euler densities, some $f(R)$ gravity theories, etc.) — see eg., ref. [23] for more details. In our approach, the absence of ghosts and tachyons is required, while it is also considered as one of the main physical selection criteria for the good higher-derivative field theories.

Another common objection against the higher-derivative gravity theories is due to the fact that all the higher-order curvature terms in the action are to be suppressed by the inverse powers of $M_{\rm Pl}$ on dimensional reasons and, therefore, they seem to be 'very small and negligible'. Though it is generically true, it does not mean that all the higher-order curvature terms are irrelevant at all scales much less than $M_{\rm Pl}$. For instance, it appears that the *quadratic* curvature terms have *dimensionless* couplings, while they can easily describe the early universe inflation (in the high-curvature regime). A non-trivial function of R in the effective gravitational action may also 'explain' *Dark Energy* in the present universe (see Sec. 11 for more).

2 Starobinsky model of inflation

It can be argued that it is the *scalar* curvature-dependent part of the gravitational effective action that is most relevant to the large-scale dynamics $H(t)$. Here are some simple arguments.

In 4 dimensions all the independent *quadratic* curvature invariants are $R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho}$, $R^{\mu\nu}R_{\mu\nu}$ and R^2. However,

$$\int d^4x\,\sqrt{-g}\,\left(R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho}-4R^{\mu\nu}R_{\mu\nu}+R^2\right) \tag{5}$$

is topological (ie. a total derivative) for any metric, while

$$\int d^4x\,\sqrt{-g}\,\left(3R^{\mu\nu}R_{\mu\nu}-R^2\right) \tag{6}$$

is also topological for any FLRW metric, because of eq. (2). Hence, the FLRW-relevant quadratically-generated gravity action is $(8\pi G_N=1)$

$$S=-\frac{1}{2}\int d^4x\,\sqrt{-g}\,\left(R-R^2/M^2\right) \tag{7}$$

[1]To the best of my knowledge, this proposal was first formulated by A.D. Sakharov in 1967.

This action is known as the (simplest) Starobinsky model [21, 22]. Its equations of motion allow a stable inflationary solution, and it is an *attractor!* When $H \gg M$, one finds

$$H \approx \left(\frac{M}{6}\right)^2 (t_{\rm end} - t) \tag{8}$$

It is the particular realization of chaotic inflation (ie. with chaotic initial conditions) [24], and with a Graceful Exit.

In the case of a generic gravitational action with the higher-order curvature terms, the Weyl dependence can be excluded due to eq. (2) again. A dependence upon the Ricci tensor can be also excluded since, otherwise, it would lead to the extra propagating massless spin-2 degree of freedom (in addition to a metric) described by the field $\partial\mathcal{L}/\partial R_{\mu\nu}$. The higher derivatives of the scalar curvature in the gravitational Lagrangian $\mathcal{L}$ just lead to more propagating scalars [25], so I simply ignore them for simplicity in what follows.

3 $f(R)$ Gravity

The Starobinsky model (7) is the special case of the $f(R)$ gravity theories [26, 27, 28] having the action

$$S_f = -\frac{1}{16\pi G_N}\int d^4x\,\sqrt{-g}\,\tilde{f}(R) \tag{9}$$

In the absence of extra matter, the gravitational (trace) equation of motion is of the fourth order with respect to the time derivative,

$$\frac{3}{a^3}\frac{d}{dt}\left(a^3\frac{d\tilde{f}'(R)}{dt}\right) + R\tilde{f}'(R) - 2\tilde{f}(R) = 0 \tag{10}$$

where we have used $H = \frac{\dot{a}}{a}$ and $R = -6(\dot{H} + 2H^2)$. The primes denote the derivatives with respect to R, and the dots denote the derivative with respect to t. Static de-Sitter solutions correspond to the roots of the equation $R\tilde{f}'(R) = 2\tilde{f}(R)$ [29].

The 00-component of the gravitational equations is of the third order with respect to the time derivative,

$$3H\frac{d\tilde{f}'(R)}{dt} - 3(\dot{H} + H^2)\tilde{f}'(R) - \frac{1}{2}\tilde{f}(R) = 0 \tag{11}$$

The (classical and quantum) *stability* conditions in $f(R)$ gravity are well known [26, 27], and are given by (in our notation)

$$\tilde{f}'(R) > 0 \qquad \text{and} \qquad \tilde{f}''(R) < 0 \tag{12}$$

respectively. The first condition (12) is needed to get a physical (non-ghost) graviton, while the second condition (12) is needed to get a physical (non-tachyonic) scalaron (see Sec. 9 for more).

Any $f(R)$ gravity is known to be classically *equivalent* to the certain scalar-tensor gravity having an (extra) propagating scalar field [30, 31, 32]. The formal equivalence can be established via a Legendre-Weyl transform.

First, the $f(R)$-gravity action (9) can be rewritten to the form

$$S_A = \frac{-1}{2\kappa^2} \int d^4x \sqrt{-g}\ \{AR - Z(A)\} \tag{13}$$

where the real scalar (or Lagrange multiplier) $A(x)$ is related to the scalar curvature R by the Legendre-like transformation:

$$R = Z'(A) \qquad \text{and} \qquad \tilde{f}(R) = RA(R) - Z(A(R)) \tag{14}$$

with $\kappa^2 = 8\pi G_N = M_{\rm Pl}^{-2}$.

Next, a Weyl transformation of the metric,

$$g_{\mu\nu}(x) \to \exp\left[\frac{2\kappa\phi(x)}{\sqrt{6}}\right] g_{\mu\nu}(x) \tag{15}$$

with arbitrary field parameter $\phi(x)$ yields

$$\sqrt{-g}\, R \to \sqrt{-g}\, \exp\left[\frac{2\kappa\phi(x)}{\sqrt{6}}\right] \left\{R - \sqrt{\frac{6}{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right)\kappa - \kappa^2 g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right\} \tag{16}$$

Therefore, when choosing

$$A(\kappa\phi) = \exp\left[\frac{-2\kappa\phi(x)}{\sqrt{6}}\right] \tag{17}$$

and ignoring a total derivative in the Lagrangian, we can rewrite the action to the form

$$\begin{aligned} S[g_{\mu\nu},\phi] = & \int d^4x \sqrt{-g}\left\{\frac{-R}{2\kappa^2} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \right. \\ & \left. + \frac{1}{2\kappa^2}\exp\left[\frac{4\kappa\phi(x)}{\sqrt{6}}\right] Z(A(\kappa\phi))\right\} \end{aligned} \tag{18}$$

in terms of the physical (and canonically normalized) scalar field $\phi(x)$, without any higher derivatives and ghosts. As a result, one arrives at the standard action of the real dynamical scalar field $\phi(x)$ *minimally* coupled to Einstein gravity and having the scalar potential

$$V(\phi) = -\frac{M_{\rm Pl}^2}{2}\exp\left\{\frac{4\phi}{M_{\rm Pl}\sqrt{6}}\right\} Z\left(\exp\left[\frac{-2\phi}{M_{\rm Pl}\sqrt{6}}\right]\right) \tag{19}$$

In the context of the inflationary theory, the *scalaron* (= scalar part of spacetime metric) ϕ can be identified with inflaton. This inflaton has clear origin, and may also be understood as the conformal mode of the metric over Minkowski or (A)dS vacuum.

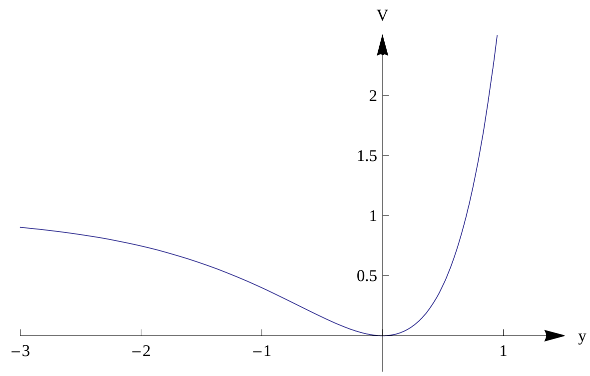

Figure 3: The inflaton scalar potential $v(x) = (e^y - 1)^2$ in the Starobinsky model, after $y \to -y$

In the Starobinsky case of $\tilde{f}(R) = R - R^2/M^2$, the inflaton scalar potential reads

$$V(y) = V_0 \left(e^{-y} - 1\right)^2 \tag{20}$$

where we have introduced the notation

$$y = \sqrt{\frac{2}{3}} \frac{\phi}{M_{\rm Pl}} \qquad \text{and} \qquad V_0 = \frac{1}{8} M_{\rm Pl}^2 M^2 \tag{21}$$

It is worth noticing here the appearance of the inflaton vacuum energy V_0 driving inflation. The end of inflation (Graceful Exit) is also clear: the scalar potential (20) has a very flat (slow-roll) ‘plateau’, ending with a ‘waterfall’ towards the minimum (Fig. 3).

It is worth emphasizing that the inflaton (scalaron) scalar potential (20) is derived here by merely assuming the existence of the R^2 term in the gravitational action. The Newton (weak gravity) limit is not applicable to early universe (including its inflationary stage), so that the dimensionless coefficient in front of the R^2 term does not have to be very small at early time. It distinguishes the primordial ‘Dark Energy’ driving inflation in the early Universe from the ‘Dark Energy’ responsible for the present universe acceleration.

4 Inflationary Theory and Observations

The *slow-roll* inflation parameters are defined by

$$\varepsilon(\phi) = \frac{1}{2} M_{\rm Pl}^2 \left(\frac{V'}{V}\right)^2 \qquad \text{and} \qquad \eta(\phi) = M_{\rm Pl}^2 \frac{V''}{V} \tag{22}$$

A necessary condition for the slow-roll approximation is the smallness of the inflation parameters

$$\varepsilon(\phi) \ll 1 \qquad \text{and} \qquad |\eta(\phi)| \ll 1 \tag{23}$$

The first condition implies $\ddot{a}(t) > 0$. The second one guarantees that inflation lasts long enough, via domination of the *friction* term in the inflaton equation of motion, $3H\dot{\phi} = -V'$.

The CMB temperature fluctuations (Fig. 2) have the scale $\delta T/T \approx 10^{-5}$ at the WMAP normalization of 500 Mpc. As is well known [15], scalar (ρ_s) and tensor (ρ_t) perturbations of metric do *decouple*. The scalar perturbations couple to the density of matter and radiation, so they are responsible for the inhomogeneities and anisotropies in the universe. The tensor perturbations (or gravity waves) also contribute to the CMB, while their experimental detection would tell us much more about inflation. The CMB raditation is expected to be *polarized* due to Compton scattering at the time of decoupling [33, 34].

The primordial spectrum is proportional to k^{n-1}, in terms of the comoving wave number k and the spectral index n, in the 2-point function (observable)

$$\left\langle \frac{\delta T(x)}{T}\frac{\delta T(y)}{T}\right\rangle \propto \int \frac{d^3k}{k^3} e^{ik(x-y)} k^{n-1} \tag{24}$$

In theory, the slope n_s of the *scalar* power spectrum, associated with the density perturbations, $\left(\frac{\delta\rho}{\rho}\right)^2 \propto k^{n_s-1}$, is given by $n_s = 1 + 2\eta - 6\varepsilon$, the slope of the *tensor* primordial spectrum, associated with gravitational waves, is $n_t = -2\varepsilon$, and the *tensor-to-scalar ratio* is $r = \delta\rho_s/\delta\rho_t = 16\varepsilon$ (see eg., ref. [15]).

It is straightforward to calculate those indices in any inflationary model with a given inflaton scalar potential. In the case of the Starobinsky model and its scalar potential (20), one finds [35, 36, 6]

$$n_s = 1 - \frac{2}{N_e} + \frac{3\ln N_e}{2N_e^2} - \frac{2}{N_e^2} + \mathcal{O}\left(\frac{\ln^2 N_e}{N_e^3}\right) \tag{25}$$

and

$$r \approx \frac{12}{N_e^2} \approx 0.004 \tag{26}$$

with $N_e \approx 55$. The very small value of r is the sharp prediction of the Starobinsky inflationary model towards r-measurements in a future.

Those theoretical values are to be compared to the observed values of the CMB radiation due to the WMAP satellite mission — see a picture of the WMAP satellite in Fig. 4. For instance, the most recent WMAP7 observations [12] yield

$$n_s = 0.963 \pm 0.012 \qquad \text{and} \qquad r < 0.24 \tag{27}$$

with the 95 % level of confidence.

The amplitude of the initial perturbations, $\Delta_R^2 = M_{\rm Pl}^4 V/(24\pi^2\varepsilon)$, is also a physical observable, whose experimental value is known due to another Cosmic Background

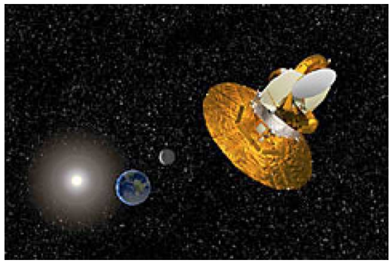

Figure 4: The WMAP satellite mission [12]

Explorer (COBE) satellite mission — see ref. [37] and Fig. 5 for a picture of the COBE satellite;

$$\left(\frac{V}{\varepsilon}\right)^{1/4} = 0.027\, M_{\rm Pl} = 6.6\times 10^{16}\ {\rm GeV} \tag{28}$$

It determines the normalization of the R^2-term in the action (7)

$$\frac{M}{M_{\rm Pl}} = 4\cdot\sqrt{\frac{2}{3}}\cdot(2.7)^2\cdot\frac{e^{-y}}{(1-e^{-y})^2}\cdot 10^{-4} \approx (3.5\pm 1.2)\cdot 10^{-6} \tag{29}$$

In particular, the inflaton mass is given by $M_{\rm inf} = M/\sqrt{6}$.

The main theoretical lessons, that we can draw from the discussion above towards our next goals, are:
(i) the main *discriminants* amongst all inflationary models are given by the values of n_s and r;
(ii) the Starobinsky model (1980) of chaotic inflation is very simple and economic. It uses gravity interactions only. It predicts the origin of inflaton and its scalar potential. It is still *viable* and *consistent* with all known observations. Inflaton is not charged

Figure 5: The COBE satellite mission [37]

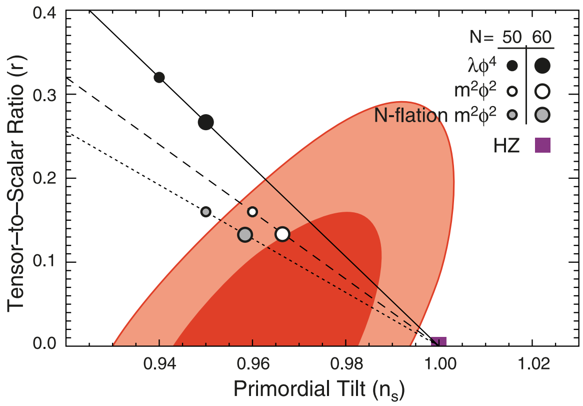

Figure 6: Starobinsky inflation vs. $m^2\phi^2/2$ and $\lambda\phi^4$

(singlet) under the SM gauge group. The Starobinsky inflation has an end (Graceful Exit), and gives the simple explanation to the WMAP-observed value of n_s. The *key* difference of Starobinsky inflation from the other standard inflationary models (having $\frac{1}{2}m^2\phi^2$ or $\lambda\phi^4$ scalar potentials) is the very low value of r — see the standard Fig. 6 for a comparison and ref. [38] for details. A discovery of primordial gravitational waves and precision measurements of the value of r (if $r \geq 0.1$) with the accuracy of 0.5% may happen due to the ongoing PLANCK satellite mission — see ref. [39] and Fig. 7 for a picture of the PLANCK satellite;

(iii) the viable inflationary models, based on $\tilde{f}(R) = R + \hat{f}(R)$ gravity, turn out to be close to the simplest Starobinsky model (over the range of R relevant to inflation), with $\hat{f}(R) \approx R^2 A(R)$ and the slowly varying function $A(R)$ in the sense

$$|A'(R)| \ll \frac{A(R)}{R} \qquad \text{and} \qquad |A''(R)| \ll \frac{A(R)}{R^2} \tag{30}$$

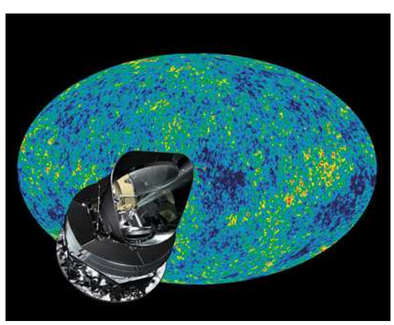

Figure 7: Ongoing Planck satellite mission [39]

5 Supergravity and Superspace

Supersymmetry (SUSY) is the symmetry between bosons and fermions. SUSY is the natural extension of Poincaré symmetry, and is well motivated in HEP beyond the SM. Supersymmetry is also needed for consistency of strings. Supergravity (SUGRA) is the theory of *local* supersymmetry that implies general coordinate invariance. In other words, considering inflation with supersymmetry necessarily leads to supergravity. As a matter of fact, most of studies of superstring- and brane-cosmology are also based on their effective description in the 4-dimensional $N = 1$ supergravity.

It is not our purpose here to give a detailed account of SUSY and SUGRA, because of the existence of several textbooks — see e.g., refs. [40, 41, 42]. In this Section I recall only the basic facts about $N = 1$ supergravity in four spacetime dimensions, which are needed for our main purposes.

A concise and manifestly supersymmetric description of SUGRA is provided by Superspace. In this section the natural units $c = \hbar = \kappa = M_{\rm Pl} = 1$ are used.

Supergravity needs a *curved* superspace. However, they are not the same, because one has to reduce the field content to the minimal ones corresponding to the off-shell supergravity multiplets. It is done by imposing certain constraints on the supertorsion tensor in curved superspace [40, 41, 42]. An off-shell supergravity multiplet has some extra (auxiliary) fields with noncanonical dimensions, in addition to physical spin-2 field (metric) and spin-3/2 field (gravitino). It is worth mentioning that imposing the off-shell constraints is independent upon writing a supergravity action.

One may work either in a full (curved) superspace or in a chiral one. There are some anvantages of using the chiral superspace, because it helps us to keep the auxiliary fields unphysical (i.e. nonpropagating).

The chiral superspace density (in the supersymmetric gauge-fixed form) reads

$$\mathcal{E}(x,\theta) = e(x)\left[1 - 2i\theta\sigma_a\bar{\psi}^a(x) + \theta^2 B(x)\right] \ , \tag{31}$$

where $e = \sqrt{-\det g_{\mu\nu}}$, $g_{\mu\nu}$ is a spacetime metric, $\psi^a_\alpha = e^a_\mu \psi^\mu_\alpha$ is a chiral gravitino, $B = S - iP$ is the complex scalar auxiliary field. We use the lower case middle greek letters $\mu, \nu, \ldots = 0, 1, 2, 3$ for curved spacetime vector indices, the lower case early latin letters $a, b, \ldots = 0, 1, 2, 3$ for flat (target) space vector indices, and the lower case early greek letters $\alpha, \beta, \ldots = 1, 2$ for chiral spinor indices.

A solution to the superspace Bianchi identities together with the constraints defining the $N = 1$ Poincaré-type *minimal* supergravity theory results in only *three* covariant tensor superfields $\mathcal{R}$, $\mathcal{G}_a$ and $\mathcal{W}_{\alpha\beta\gamma}$, subject to the off-shell relations [40, 41, 42]:

$$\mathcal{G}_a = \bar{\mathcal{G}}_a \ , \qquad \mathcal{W}_{\alpha\beta\gamma} = \mathcal{W}_{(\alpha\beta\gamma)} \ , \qquad \bar{\nabla}_{\dot{\alpha}}\mathcal{R} = \bar{\nabla}_{\dot{\alpha}}\mathcal{W}_{\alpha\beta\gamma} = 0 \ , \tag{32}$$

and

$$\bar{\nabla}^{\dot{\alpha}}\mathcal{G}_{\alpha\dot{\alpha}} = \nabla_\alpha\mathcal{R} \ , \qquad \nabla^\gamma\mathcal{W}_{\alpha\beta\gamma} = \tfrac{i}{2}\nabla_\alpha{}^{\dot{\alpha}}\mathcal{G}_{\beta\dot{\alpha}} + \tfrac{i}{2}\nabla_\beta{}^{\dot{\alpha}}\mathcal{G}_{\alpha\dot{\alpha}} \ , \tag{33}$$

where $(\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}.\nabla_{\alpha\dot{\alpha}})$ stand for the curved superspace $N = 1$ supercovariant derivatives, and the bars denote complex conjugation.

The covariantly chiral complex scalar superfield $\mathcal{R}$ has the scalar curvature R as the coefficient at its θ^2 term, the real vector superfield $\mathcal{G}_{\alpha\dot{\alpha}}$ has the traceless Ricci tensor, $R_{\mu\nu}+R_{\nu\mu}-\frac{1}{2}g_{\mu\nu}R$, as the coefficient at its $\theta\sigma^a\bar{\theta}$ term, whereas the covariantly chiral, complex, totally symmetric, fermionic superfield $\mathcal{W}_{\alpha\beta\gamma}$ has the self-dual part of the Weyl tensor $C_{\alpha\beta\gamma\delta}$ as the coefficient at its linear θ^δ-dependent term.

A generic Lagrangian representing the supergravitational effective action in (full) superspace, reads

$$\mathcal{L}=\mathcal{L}(\mathcal{R},\mathcal{G},\mathcal{W},\ldots) \tag{34}$$

where the dots stand for arbitrary supercovariant derivatives of the superfields.

The Lagrangian (34) it its most general form is, however, unsuitable for physical applications, not only because it is too complicated, but just because it generically leads to propagating auxiliary fields, which break the balance of the bosonic and fermionic degrees of freedom. The important physical condition of keeping the supergravity auxiliary fields to be truly auxiliary (ie. nonphysical or nonpropagating) in field theories with the higher derivatives was dubbed the *'auxiliary freedom'* in refs. [43, 44]. To get the supergravity actions with the 'auxiliary freedom', we will use a chiral (curved) superspace.

6 $F(\mathcal{R})$ Supergravity in Superspace

Let us first concentrate on the scalar-curvature-sector of a generic higher-derivative supergravity (34), which is most relevant to the FRLW cosmology, by ignoring the tensor curvature superfields $\mathcal{W}_{\alpha\beta\gamma}$ and $\mathcal{G}_{\alpha\dot{\alpha}}$, as well as the derivatives of the scalar superfield $\mathcal{R}$, like that in Sec. 2. Then we arrive at the chiral superspace action

$$S_F=\int d^4x d^2\theta\, \mathcal{E}F(\mathcal{R})+\mathrm{H.c.} \tag{35}$$

governed by a chiral or *holomorphic* function $F(\mathcal{R})$. [2] Besides having the manifest local $N=1$ supersymmetry, the action (35) has the auxiliary freedom since the auxiliary field B does not propagate. It distinguishes the action (35) from other possible truncations of eq. (34). The action (35) gives rise to the spacetime *torsion* generated by gravitino, while its bosonic terms have the form

$$S_f=-\frac{1}{2}\int d^4x\, \sqrt{-g}\, \tilde{f}(R) \tag{36}$$

Hence, eq. (35) can also be considered as the locally $N=1$ supersymmetric extension of the $f(R)$-type gravity (Sec. 3). However, in the context of supergravity, the 'supersymmetrizable' bosonic functions $\tilde{f}(R)$ are very restrictive (see Secs. 9 and 10).

[2]The similar component field construction, by the use of the 4D, $N=1$ superconformal tensor calculus, was given in ref. [45].

The superfield action (35) is classically equivalent to

$$S_V = \int d^4x d^2\theta\, \mathcal{E}\, [\mathcal{Z}\mathcal{R} - V(\mathcal{Z})] + \text{H.c.} \tag{37}$$

with the covariantly chiral superfield $\mathcal{Z}$ as the Lagrange multiplier superfield. Varying the action (37) with respect to $\mathcal{Z}$ gives back the original action (35) provided that

$$F(\mathcal{R}) = \mathcal{R}\mathcal{Z}(\mathcal{R}) - V(\mathcal{Z}(\mathcal{R})) \tag{38}$$

where the function $\mathcal{Z}(\mathcal{R})$ is defined by inverting the function

$$\mathcal{R} = V'(\mathcal{Z}) \tag{39}$$

Equations (38) and (39) define the superfield Legendre transform, and imply

$$F'(\mathcal{R}) = Z(\mathcal{R}) \qquad \text{and} \qquad F''(\mathcal{R}) = Z'(\mathcal{R}) = \frac{1}{V''(\mathcal{Z}(\mathcal{R}))} \tag{40}$$

where $V'' = d^2V/d\mathcal{Z}^2$. The second formula (40) is the *duality* relation between the supergravitational function F and the chiral superpotential V.

A supersymmetric (local) Weyl transform of the acton (37) can be done entirely in superspace. In terms of the field components, the super-Weyl transform amounts to a Weyl transform, a chiral rotation and a (superconformal) S-supersymmetry transformation [46]. The chiral density superfield $\mathcal{E}$ appears to be the chiral *compensator* of the super-Weyl transformations,

$$\mathcal{E} \to e^{3\Phi}\mathcal{E} \tag{41}$$

whose parameter Φ is an arbitrary covariantly chiral superfield, $\bar{\nabla}_{\dot{\alpha}}\Phi = 0$. Under the transformation (41) the covariantly chiral superfield $\mathcal{R}$ transforms as

$$\mathcal{R} \to e^{-2\Phi}\left(\mathcal{R} - \tfrac{1}{4}\bar{\nabla}^2\right)e^{\bar{\Phi}} \tag{42}$$

The super-Weyl chiral superfield parameter Φ can be traded for the chiral Lagrange multiplier $\mathcal{Z}$ by using a generic gauge condition

$$\mathcal{Z} = \mathcal{Z}(\Phi) \tag{43}$$

where $\mathcal{Z}(\Phi)$ is a holomorphic function of Φ. It results in the action

$$S_\Phi = \int d^4x d^4\theta\, E^{-1} e^{\Phi+\bar{\Phi}}\, [\mathcal{Z}(\Phi) + \text{H.c.}] - \int d^4x d^2\theta\, \mathcal{E} e^{3\Phi} V(\mathcal{Z}(\Phi)) + \text{H.c.} \tag{44}$$

Equation (44) has the *standard* form of the action of a chiral matter superfield coupled to supergravity,

$$S[\Phi,\bar{\Phi}] = \int d^4x d^4\theta\, E^{-1}\Omega(\Phi,\bar{\Phi}) + \left[\int d^4x d^2\theta\, \mathcal{E} P(\Phi) + \text{H.c.}\right] \tag{45}$$

in terms of the non-chiral potential $\Omega(\Phi,\bar{\Phi})$ and the chiral superpotential $P(\Phi)$. In our case (44) we find

$$\Omega(\Phi,\bar{\Phi}) = e^{\Phi+\bar{\Phi}}\left[\mathcal{Z}(\Phi)+\bar{\mathcal{Z}}(\bar{\Phi})\right] \ , \quad P(\Phi) = -e^{3\Phi}V(\mathcal{Z}(\Phi)) \tag{46}$$

The *Kähler* potential $K(\Phi,\bar{\Phi})$ is given by

$$K = -3\ln(-\frac{\Omega}{3}) \quad \text{or} \quad \Omega = -3e^{-K/3} \tag{47}$$

so that the action (45) is invariant under the supersymmetric (local) Kähler-Weyl transformations

$$K(\Phi,\bar{\Phi}) \to K(\Phi,\bar{\Phi}) + \Lambda(\Phi) + \bar{\Lambda}(\bar{\Phi}) \ , \qquad P(\Phi) \to -e^{-\Lambda(\Phi)}P(\Phi) \tag{48}$$

with the chiral superfield parameter $\Lambda(\Phi)$. It follows that

$$\mathcal{E} \to e^{\Lambda(\Phi)}\mathcal{E} \tag{49}$$

The scalar potential in terms of the usual fields is given by the standard formula [47]

$$\mathcal{V}(\phi,\bar{\phi}) = \left. e^{K}\left\{\left|\tfrac{\partial P}{\partial\Phi}+\tfrac{\partial K}{\partial\Phi}P\right|^2 - 3\left|P\right|^2\right\}\right| \tag{50}$$

where all the superfields are restricted to their leading field components, $\Phi| = \phi(x)$, and we have introduced the notation

$$\left|\frac{\partial P}{\partial\Phi}+\frac{\partial K}{\partial\Phi}P\right|^2 \equiv \left|D_\Phi P\right|^2 = D_\Phi P(K^{-1}_{\Phi\bar{\Phi}})\bar{D}_{\bar{\Phi}}\bar{P} \tag{51}$$

with $K_{\Phi\bar{\Phi}} = \partial^2 K/\partial\Phi\partial\bar{\Phi}$. Equation (50) can be simplified by making use of the Kähler-Weyl invariance (48) that allows one to choose a gauge

$$P = 1 \tag{52}$$

It is equivalent to the well known fact that the scalar potential (50) is actually governed by the *single* (Kähler-Weyl-invariant) potential

$$G(\Phi,\bar{\Phi}) = \Omega + \ln\left|P\right|^2 \tag{53}$$

In our case (46) we find

$$G = e^{\Phi+\bar{\Phi}}\left[\mathcal{Z}(\Phi)+\bar{\mathcal{Z}}(\bar{\Phi})\right] + 3(\Phi+\bar{\Phi}) + \ln(V(\mathcal{Z}(\Phi)) + \ln(\bar{V}(\bar{\mathcal{Z}}(\bar{\Phi})) \tag{54}$$

So let us choose a gauge by the condition

$$3\Phi + \ln(V(\mathcal{Z}(\Phi)) = 0 \quad \text{or} \quad V(\mathcal{Z}(\Phi)) = e^{-3\Phi} \tag{55}$$

that is equivalent to eq. (52). Then the G-potential (54) gets simplified to

$$G = e^{\Phi+\bar{\Phi}}\left[\mathcal{Z}(\Phi)+\bar{\mathcal{Z}}(\bar{\Phi})\right] \tag{56}$$

There is the correspondence between a holomorphic function $F(\mathcal{R})$ in the supergravity action (35) and a holomorphic function $\mathcal{Z}(\Phi)$ defining the scalar potential (50),

$$\mathcal{V} = e^{G}\left[\left(\frac{\partial^2 G}{\partial\Phi\partial\bar{\Phi}}\right)^{-1}\frac{\partial G}{\partial\Phi}\frac{\partial G}{\partial\bar{\Phi}} - 3\right]\Bigg| \tag{57}$$

in the classically equivalent scalar-tensor supergravity.

To the end of this section, I would like to comment on the standard way of the inflationary model building by a choice of $K(\Phi,\bar{\Phi})$ and $P(\Phi)$ — see eg., refs. [48, 49] for a review.

The factor $\exp(K/M_{\rm Pl}^2)$ in the F-type scalar potential (50) of the chiral matter-coupled supergravity, in the case of the *canonical* Kähler potential, $K \propto \overline{\Phi}\Phi$, results in the scalar potential $V \propto \exp(|\Phi|^2/M_{\rm Pl}^2)$ that is too steep to support chaotic inflation. Actually, it also implies $\eta \approx 1$ or, equivalently, $M^2_{\rm inflaton} \approx V_0/M^2_{\rm Pl} \approx H^2$. It is known as the *$\eta$-problem* in supergravity [50].

As is clear from our discussion above, the η-problem is not really a supergravity problem, but it is the problem associated with the choice of the canonical Kähler potential for an inflaton superfield. The Kähler potential in supergravity is a (Kähler) gauge-dependent quantity, and its quantum renormalization is not under control. Unlike the one-field inflationary models, a generic Kähler potential is a function of at least two real scalar fields, so it implies a nonvanishing curvature in the target space of the *non-linear sigma-model* associated with the Kähler kinetic term. [3] Hence, a generic Kähler potential cannot be brought to the canonical form by a field redefinion.

To solve the η-problem associated with the simplest (naive) choice of the Kähler potential, on may assume that the Kähler potential K possesses some shift symmetries (leading to its *flat* directions), and then choose inflaton in one such flat direction [52]. However, in order to get inflation that way, one also has to add ("by hand") the proper inflaton superpotential breaking the initially introduced shift symmetry, and then stabilize the inflationary trajectory with the help of yet another matter superfield.

The possible alternative is the D-term mechanism [53], where the inflaton particle belongs to the matter *gauge* sector and, as a result, inflation is highly sensitive to gauge charges [53].

It is worth mentioning that in the (perturbative) superstring cosmology one gets the Kähler potential (see e.g., refs. [54, 55])

$$K \propto \log(\text{moduli polynomial})_{\rm CY} \tag{58}$$

over a Calabi-Yau (CY) space in the type-IIB superstring compactification, thus avoiding the η-problem but leading to a plenty of choices ("embarrassment of riches") in the String Landscape.

[3] See eg., ref. [51] for more about the non-linear sigma-models.

Finally, one still has to accomplish stability of a given inflationary model in supergravity against quantum corrections. Such corrections can easily spoil the flatness of the inflaton potential. The Kähler kinetic term is not protected against quantum corrections, because it is given by a full superspace integral (unlike the chiral superpotential term). The $F(\mathcal{R})$ supergravity action (35) is given by a *chiral* superspace integral, so that it is protected against the quantum corrections given by full superspace integrals.

To conclude this section, we claim that an $N=1$ locally supersymmetric extension of $f(R)$ gravity is possible. It is non-trivial because the auxiliary freedom has to be preserved. The new supergravity action (35) is classically equivalent to the standard $N=1$ Poincaré supergravity coupled to a *dynamical* chiral matter superfield, whose Kähler potential and the superpotential are dictated by a single *holomorphic* function. Inflaton can be identified with the real scalar field component of that chiral matter superfield originating from the supervielbein, and thus has the geometrical origin.

It is worth noticing that the action (35) allows a natural extension in chiral curved superspace, due to the last equation (32), namely,

$$S_{\rm ext} = \int d^4x d^2\theta\, \mathcal{E} F(\mathcal{R}, \mathcal{W}^2) + {\rm H.c.} \tag{59}$$

where $\mathcal{W}_{\alpha\beta\gamma}$ is the $N=1$ covariantly-chiral Weyl superfield of the $N=1$ superspace supergravity, and $\mathcal{W}^2 = \mathcal{W}_{\alpha\beta\gamma}\mathcal{W}^{\alpha\beta\gamma}$. The action (59) also has the auxiliary freedom. In Supersring Theory, the Weyl-tensor-dependence of the gravitational effective action is unambigously determined by the superstring scattering amplitudes or by the super-Weyl invariance of the corresponding non-linear sigma-model (see eg., ref. [51]).

A possible connection of $F(\mathcal{R})$ supergravity to the *Loop Quantum Gravity* was investigated in ref. [3].

7 No-scale $F(\mathcal{R})$ Supergravity

In this section we would like to investigate a possibility of spontaneous supersymmetry breaking, without fine tuning, by imposing the condition of the vanishing scalar potential. Those no-scale supergravities are the starting point of many phenomenological applications of supergravity in HEP and inflationary theory, including string theory applications — see eg., refs. [56, 57] and references therein.

The no-scale supergravity arises by demanding the scalar potential (50) to vanish. It results in the vanishing cosmological constant without fine-tuning [58]. The no-scale supergravity potential G has to obey the non-linear 2nd-order partial differential equation, which follows from eq. (57),

$$3\frac{\partial^2 G}{\partial\Phi\partial\bar{\Phi}} = \frac{\partial G}{\partial\Phi}\frac{\partial G}{\partial\bar{\Phi}} \tag{60}$$

A gravitino mass $m_{3/2}$ is given by the vacuum expectation value [41]

$$m_{3/2} = \left\langle e^{G/2}\right\rangle \tag{61}$$

so that the spontaneous supersymmetry breaking scale can be chosen at will.

The well known exact solution to eq. (60) is given by

$$G = -3\log(\Phi+\bar{\Phi}) \tag{62}$$

In the recent literature, the no-scale solution (62) is usually modified by other terms, in order to achieve the universe with a positive cosmological constant — see e.g., the KKLT mechanism [59].

To appreciate the difference between the standard no-scale supergravity solution and our 'modified' supergravity, it is worth noticing that the Ansatz (62) is not favoured by our potential (56). In our case, demanding eq. (60) gives rise to the 1st-order non-linear partial differential equation

$$3\left(e^{\bar{\Phi}}X' + e^{\Phi}\bar{X}'\right) = \left|e^{\bar{\Phi}}X' + e^{\Phi}\bar{X}\right|^2 \tag{63}$$

where we have introduced the notation

$$\mathcal{Z}(\Phi) = e^{-\Phi}X(\Phi)\ , \qquad X' = \frac{dX}{d\Phi} \tag{64}$$

in order to get the differential equation in its most symmetric and concise form.

Accordingly, the gravitino mass (61) is given by

$$m_{3/2} = \left\langle \exp\frac{1}{2}\left(e^{\bar{\Phi}}X + e^{\Phi}\bar{X}\right)\right\rangle \tag{65}$$

I am not aware of any non-trivial holomorphic exact solution to eq. (63). However, should it obey a holomorphic differential equation of the form

$$X' = e^{\Phi}g(X,\Phi) \tag{66}$$

with a holomorphic function $g(X,\Phi)$, eq. (63) gives rise to the functional equation

$$3\,(g+\bar{g}) = \left|e^{\bar{\Phi}}g + \bar{X}\right|^2 \tag{67}$$

Being restricted to the real variables $\Phi=\bar{\Phi}\equiv y$ and $X=\bar{X}\equiv x$, eq. (63) reads

$$6x' = e^{y}(x'+x)^2\ , \quad \text{where} \quad x' = \frac{dx}{dy} \tag{68}$$

This equation can be integrated after a change of variables,

$$x = e^{-y}u\ , \tag{69}$$

and it leads to a quadratic equation with respect to $u' = du/dy$,

$$(u')^2 - 6u' + 6u = 0 \tag{70}$$

It follows

$$y = \int^u \frac{d\xi}{3\pm\sqrt{3(3-2\xi)}} = \mp\sqrt{1-\tfrac{2}{3}u} + \ln\left(\sqrt{3(3-2u)}\pm 3\right) + C\ . \tag{71}$$

8 Fields from Superfields in $F(\mathcal{R})$ Supergravity

For simplicity, we now set all fermionic fields to zero, and keep only bosonic field components of the superfields. It greatly simplies all equations but makes supersymmetry to be manifestly broken. Of course, SUSY is restored after adding back all the fermionic terms.

Applying the standard superspace chiral density formula [40, 41, 42]

$$\int d^4x d^2\theta\, \mathcal{E}\mathcal{L} = \int d^4x\, e \left\{ \mathcal{L}_{\rm last} + B\mathcal{L}_{\rm first} \right\} \tag{72}$$

to the action (35) yields its bosonic part in the form

$$(-g)^{-1/2}L_{\rm bos} \equiv f(R,\tilde{R};X,\bar{X}) = F'(\bar{X})\left[\tfrac{1}{3}R_* + 4\bar{X}X\right] + 3XF(\bar{X}) + {\rm H.c.} \tag{73}$$

where the primes denote differentiation with respect to the given argument. We have used the notation

$$X = \tfrac{1}{3}B \qquad \text{and} \qquad R_* = R + \frac{i}{2}\varepsilon^{abcd}R_{abcd} \equiv R + i\tilde{R} \tag{74}$$

The $\tilde{R}$ does not vanish in $F(\mathcal{R})$ supergravity, and it represents the *axion* field that is the pseudo-scalar superpartner of real scalaron field in our construction.

Varying eq. (73) with respect to the auxiliary fields X and $\bar{X}$,

$$\frac{\partial L_{\rm bos}}{\partial X} = \frac{\partial L_{\rm bos}}{\partial \bar{X}} = 0 \tag{75}$$

gives rise to the algebraic equations on the auxiliary fields,

$$3\bar{F} + X(4\bar{F}' + 7F') + 4\bar{X}XF'' + \tfrac{1}{3}F''R_* = 0 \tag{76}$$

and its conjugate

$$3F + \bar{X}(4F' + 7\bar{F}') + 4\bar{X}X\bar{F}'' + \tfrac{1}{3}\bar{F}''\bar{R}_* = 0 \tag{77}$$

where $F = F(X)$ and $\bar{F} = \bar{F}(\bar{X})$. The algebraic equations (76) and (77) cannot be explicitly solved for X in a generic $F(\mathcal{R})$ supergravity.

To recover the standard (pure) supergravity in our approach, let us consider the simple special case when

$$F'' = 0 \qquad \text{or, equivalently,} \qquad F(\mathcal{R}) = f_0 - \tfrac{1}{2}f_1\mathcal{R} \tag{78}$$

with some complex constants f_0 and f_1, where ${\rm Re}f_1 > 0$. Then eq. (76) is easily solved as

$$\bar{X} = \frac{3f_0}{5({\rm Re}f_1)} \tag{79}$$

Substituting this solution back into the Lagrangian (73) yields

$$L = -\tfrac{1}{3}(\mathrm{Re} f_1)R + \frac{9\,|f_0|^2}{5(\mathrm{Re} f_1)} \equiv -\tfrac{1}{2}M_{\mathrm{Pl}}^2 R - \Lambda \tag{80}$$

where we have introduced the reduced Planck mass M_{Pl}, and the cosmological constant Λ as

$$\mathrm{Re} f_1 = \tfrac{3}{2}M_{\mathrm{Pl}}^2 \qquad \text{and} \qquad \Lambda = \frac{-6\,|f_0|^2}{5M_{\mathrm{Pl}}^2} \tag{81}$$

It is the standard pure supergravity with a *negative* cosmological constant [40, 41, 42].

9 Generic $\mathcal{R}^2$ supergravity, and AdS Bound

The simplest non-trivial $F(\mathcal{R})$ supergravity is obtained by choosing $F'' = const. \neq 0$ that leads to the $\mathcal{R}^2$-supergravity defined by a generic *quadratic* polynomial in terms of the scalar supercurvature [8].

Let us recall that the stability conditions in $f(R)$-gravity are given by eqs. (12) in the notation (9). In the notation (73) used here, ie. when $f(R) = -\frac{1}{2}M_{\mathrm{Pl}}^2\tilde{f}(R)$, one gets the opposite signs,

$$f'(R) < 0 \tag{82}$$

and

$$f''(R) > 0 \tag{83}$$

The first (classical stability) condition (82) is related to the sign factor in front of the Einstein-Hilbert term (linear in R) in the $f(R)$-gravity action, and it ensures that graviton is not a ghost. The second (quantum stability) condition (83) guarantees that scalaron is not a tachyon.

Being mainly interested in the inflaton part of the bosonic $f(R)$-gravity action that follows from eq. (73), we set both gravitino and axion to zero, which also implies $R_* = R$ and a *real* X.

In $F(R)$ supergravity the stability condition (82) is to be replaced by a stronger condition,

$$F'(X) < 0 \tag{84}$$

It is easy to verify that eq. (82) follows from eq. (84) because of eq. (75). Equation (84) also ensures the classical stability of the bosonic $f(R)$ gravity embedding into the full $F(\mathcal{R})$ supergravity against small fluctuations of the axion field.

In this Section we investigate the most general Ansatz (with $F'' = const. \neq 0$) that leads to the simplest non-trivial toy-model of $F(R)$ supergravity with the master function

$$F(\mathcal{R}) = f_0 - \frac{1}{2}f_1\mathcal{R} + \frac{1}{2}f_2\mathcal{R}^2 \tag{85}$$

having three coupling constants f_0, f_1 and f_2. We take all of them to be real, since we ignore this potential source of CP-violation here (see, however, the Outlook in Sec. 15). As regards the mass dimensions of the quantities introduced, we have

$$[F] = [f_0] = 3\ , \quad [R] = [f_1] = 2\ , \quad \text{and} \quad [\mathcal{R}] = [f_2] = 1 \tag{86}$$

The bosonic Lagrangian (73) with the function (85) reads

$$(-g)^{-1/2} L_{\rm bos} = 11 f_2 X^3 - 7 f_1 X^2 + \left(\tfrac{2}{3} f_2 R + 6 f_0\right) X - \tfrac{1}{3} f_1 R \tag{87}$$

Hence, the auxiliary field equation (75) takes the form of a *quadratic* equation,

$$\tfrac{33}{2} f_2 X^2 - 7 f_1 X + \tfrac{1}{3} R f_2 + 3 f_0 = 0 \tag{88}$$

whose solution is given by

$$X_{\pm} = \frac{7}{3\cdot 11}\left[\frac{f_1}{f_2} \pm \sqrt{\frac{2\cdot 11}{7^2}(R_{\rm max} - R)}\right] \tag{89}$$

where we have introduced the *maximal* scalar curvature

$$R_{\rm max} = \frac{7^2}{2\cdot 11}\frac{f_1^2}{f_2^2} - 3^2 \frac{f_0}{f_2} \tag{90}$$

Equation (89) obviously implies the automatic bound on the scalar curvature (from one side only). In our notation, it corresponds to the (AdS) bound on the scalar curvature from above,

$$R < R_{\rm max} \tag{91}$$

The existence of the built-in maximal (upper) scalar curvature (or the AdS bound) is a nice bonus of our construction. It is similar to the factor $\sqrt{1 - v^2/c^2}$ in Special Relativity. Yet another close analogy comes from the *Born-Infeld* non-linear extension of Maxwell electrodynamics, whose (dual) Hamiltonian is proportional to [51]

$$\left(1 - \sqrt{1 - \vec{E}^2/E^2_{\rm max} - \vec{H}^2/H^2_{\rm max} + (\vec{E}\times\vec{H})^2/E^2_{\rm max}H^2_{\rm max}}\right) \tag{92}$$

in terms of the electric and magnetic fields $\vec{E}$ and $\vec{H}$, respectively, with their maximal values. For instance, in String Theory one has $E_{\rm max} = H_{\rm max} = (2\pi\alpha')^{-1}$ [51].

Substituting the solution (89) back into eq. (87) yields the corresponding $f(R)$-gravity Lagrangian

$$\begin{aligned} f_{\pm}(R) =& \frac{2\cdot 7}{11}\frac{f_0 f_1}{f_2} - \frac{2\cdot 7^3}{3^3\cdot 11^2}\frac{f_1^3}{f_2^2} \\ & - \frac{19}{3^2\cdot 11} f_1 R \mp \sqrt{\frac{2}{11}}\left(\frac{2^2}{3^3} f_2\right)(R_{\rm max} - R)^{3/2} \end{aligned} \tag{93}$$

Expanding eq. (93) into power series of R yields

$$f_\pm(R) = -\Lambda_\pm - a_\pm R + b_\pm R^2 + \mathcal{O}(R^3) \tag{94}$$

whose coefficients are given by

$$\Lambda_\pm = \frac{2\cdot 7}{3^2\cdot 11} f_1 \left(R_{\rm max} - \frac{7^2}{2\cdot 3\cdot 11}\frac{f_1^2}{f_2^2}\right) \pm \sqrt{\frac{2}{11}}\left(\frac{2^2}{3^3}f_2\right) R_{\rm max}^{3/2} \tag{95}$$

$$a_\pm = \frac{19}{3^2\cdot 11} f_1 \mp \sqrt{\frac{2}{11}R_{\rm max}}\left(\frac{2}{3^2}f_2\right) \tag{96}$$

and

$$b_\pm = \mp\sqrt{\frac{2}{11 R_{\rm max}}}\left(\frac{f_2}{2\cdot 3^2}\right) \tag{97}$$

Those equations greatly simplify when $f_0 = 0$. One finds [5, 8]

$$f_\pm^{(0)}(R) = \frac{-5\cdot 17 M_{\rm Pl}^2}{2\cdot 3^2\cdot 11} R + \frac{2\cdot 7}{3^2\cdot 11} M_{\rm Pl}^2 \left(R - R_{\rm max}\right)\left[1 \pm \sqrt{1 - R/R_{\rm max}}\,\right] \tag{98}$$

where we have chosen

$$f_1 = \frac{3}{2} M_{\rm Pl}^2 \tag{99}$$

in order to get the standard normalization of the Einstein-Hilbert term that is linear in R. Then, in the limit $R_{\rm max} \to +\infty$, both functions $f_\pm^{(0)}(R)$ reproduce General Relativity. In another limit $R \to 0$, one finds a *vanishing* or *positive* cosmological constant,

$$\Lambda_-^{(0)} = 0 \qquad \text{and} \qquad \Lambda_+^{(0)} = \frac{2^2\cdot 7}{3^2\cdot 11} M_{\rm Pl}^2 R_{\rm max} \tag{100}$$

The stability conditions are given by eqs. (82), (83) and (84), while the 3rd condition implies the 2nd one. In our case (93) we have

$$f'_\pm(R) = -\frac{19}{3^2\cdot 11} f_1 \pm \sqrt{\frac{2}{11}}\left(\frac{2}{3^2}f_2\right)\sqrt{R_{\rm max} - R}\, < 0 \tag{101}$$

and

$$f''_\pm(R) = \mp\left(\frac{f_2}{3^2}\right)\sqrt{\frac{2}{11(R_{\rm max}-R)}}\, > 0 \tag{102}$$

while eqs. (84), (85) and (89) yield

$$\pm\sqrt{\frac{2\cdot 11}{7^2}(R_{\rm max} - R)}\, < \frac{19}{2\cdot 7}\frac{f_1}{f_2} \tag{103}$$

It follows from eq. (102) that

$$f_2^{(+)} < 0 \qquad \text{and} \qquad f_2^{(-)} > 0 \tag{104}$$

Then the stability condition (83) is obeyed for any value of R.

As regards the $(-)$*-case*, there are *two* possibilities depending upon the sign of f_1. Should f_1 be *positive*, all the remaining stability conditions are automatically satisfied, ie. in the case of both $f_2^{(-)} > 0$ and $f_1^{(-)} > 0$.

Should f_1 be *negative*, $f_1^{(-)} < 0$, we find that the remaining stability conditions (101) and (103) are *the same*, as they should, while they are both given by

$$R < R_{\max} - \frac{19^2}{2^3 \cdot 11} \frac{f_1^2}{f_2^2} = -\frac{3 \cdot 5}{2^3 \cdot 11} \frac{f_1^2}{f_2^2} - 3^2 \frac{f_0}{f_2} \equiv R_{\max}^{\rm ins} \tag{105}$$

As regards the $(+)$*-case*, eq. (103) implies that f_1 should be *negative*, $f_1 < 0$, whereas then eqs. (101) and (103) result in *the same* condition (105) again.

Since $R_{\max}^{\rm ins} < R_{\max}$, our results imply that the instability happens *before* R reaches $R_{\max}$ in all cases with negative f_1.

As regards the particularly simple case (98), the stability conditions allow us to choose the lower sign only.

A different example arises with a negative f_1. When choosing the lower sign (ie. a positive f_2) for definiteness, we find

$$\begin{aligned} f_-(R) = &-\frac{2 \cdot 7}{11} f_0 \left| \frac{f_1}{f_2} \right| + \frac{2 \cdot 7^3}{3^3 \cdot 11^2} \left| \frac{f_1^3}{f_2^2} \right| \\ &+ \frac{19}{3^2 \cdot 11} \left| f_1 \right| R + \sqrt{\frac{2}{11}} \left(\frac{2^2}{3^3} f_2 \right) (R_{\max} - R)^{3/2} \end{aligned} \tag{106}$$

Demanding the standard normalization of the Einstein-Hilbert term in this case implies

$$R_{\max} = \frac{3^4 \cdot 11}{2^3 f_2^2} \left(\frac{M_{\rm Pl}^2}{2} + \frac{19}{3^2 \cdot 11} \left| f_1 \right| \right)^2 \tag{107}$$

where we have used eq. (96). It is easy to verify by using eq. (95) that the cosmological constant is always *negative* in this case, and the instability bound (105) is given by

$$R_{\max}^{\rm ins} = \frac{3^4 \cdot 11 M_{\rm Pl}^2}{2^3 f_2^2} \left(\frac{M_{\rm Pl}^2}{2^2} + \frac{19 \left| f_1 \right|}{3^2 \cdot 11} \right) < R_{\max} \tag{108}$$

The $f_-(R)$ function of eq. (93) can be rewritten to the form

$$f(R) = \frac{7^3}{3^3 \cdot 11^2} \frac{f_1^3}{f_2^2} - \frac{2 \cdot 7}{3^2 \cdot 11} f_1 R_{\max} - \frac{19}{3^2 \cdot 11} f_1 R + f_2 \sqrt{\frac{2^5}{3^6 \cdot 11}} (R_{\max} - R)^{3/2} \tag{109}$$

where we have used eq. (90). There are *three* physically different regimes:

(i) the *high-curvature regime*, $R < 0$ and $|R| \gg R_{\max}$. Then eq. (109) implies

$$f(R) \approx -\Lambda_h - a_h R + c_h \left| R \right|^{3/2} \tag{110}$$

whose coefficients are given by

$$\begin{aligned}
\Lambda_h &= \frac{2 \cdot 7}{3^2 \cdot 11} f_1 R_{\max} - \frac{7^3}{3^3 \cdot 11^2} \frac{f_1^3}{f_2^2} \ , \\
a_h &= \frac{19}{3^2 \cdot 11} f_1 \ , \\
c_h &= \sqrt{\frac{2}{11}} \left(\frac{2^2}{3^3} f_2 \right)
\end{aligned} \tag{111}$$

(ii) the *low-curvature regime,* $|R/R_{\max}| \ll 1$. Then eq. (109) implies

$$f(R) \approx -\Lambda_l - a_l R \ , \tag{112}$$

whose coefficients are given by

$$\begin{aligned}
\Lambda_l &= \Lambda_h - \sqrt{\frac{2R^3_{\max}}{11}} \left(\frac{2^2}{3^3} f_2 \right) \ , \\
a_l &= a_h + \sqrt{\frac{2R_{\max}}{11}} \left(\frac{2}{3^2} f_2 \right) = a_- = \frac{M^2_{\mathrm{Pl}}}{2} \ ,
\end{aligned} \tag{113}$$

where we have used eq. (96).

(iii) the *near-the-bound regime* (assuming that no instability happens before it), $R = R_{\max} + \delta R$, $\delta R < 0$, and $|\delta R/R_{\max}| \ll 1$. Then eq. (109) implies

$$f(R) \approx -\Lambda_b + a_b \, |\delta R| + c_b \, |\delta R|^{3/2} \tag{114}$$

whose coefficients are

$$\begin{aligned}
\Lambda_b &= \frac{1}{3} f_1 R_{\max} - \frac{7^3}{3^3 \cdot 11^2} \frac{f_1^3}{f_2^2} \ , \\
a_b &= a_h \ , \\
c_b &= \sqrt{\frac{2}{11}} \left(\frac{2^2}{3^3} f_2 \right)
\end{aligned} \tag{115}$$

The cosmological dynamics may be either directly derived from the gravitational equations of motion in the $f(R)$-gravity with a given function $f(R)$, or just read off from the form of the corresponding scalar potential of a scalaron (see below). For instance, as was demonstrated in ref. [5] for the special case $f_0 = 0$, a cosmological expansion is possible in the regime (i) towards the regime (ii), and then, perhaps, to the regime (iii) unless an instability occurs.

However, one should be careful since our toy-model (85) does not pretend to be viable in the low-curvature regime, eg., for the present Universe. Nevertheless, if one wants to give it some physical meaning there, by identifying it with General Relativity,

then one should also fine-tune the cosmological constant Λ_l in eq. (113) to be "small" and positive. We find that it amounts to

$$R_{\max} \approx \frac{3^4 \cdot 7^2 \cdot 11}{2^5 \cdot 19^2} \frac{M_{\rm Pl}^4}{f_2^2} \equiv R_{\Lambda=0} \tag{116}$$

with the actual value of $R_{\max}$ to be "slightly" above of that bound, $R_{\max} > R_{\Lambda=0}$. It is also posssible to have the vanishing cosmological constant, $\Lambda_l = 0$, when choosing $R_{\max} = R_{\Lambda=0}$. It is worth mentioning that it relates the values of $R_{\max}$ and f_2.

The particular $\mathcal{R}^2$-supergravity model (with $f_0 = 0$) was introduced in ref. [5] in an attempt to get viable embedding of the Starobinsky model into $F(\mathcal{R})$-supergravity. However, it failed because, as was found in ref. [5], the higher-order curvature terms cannot be ignored in eq. (98), ie. the R^n-terms with $n \geq 3$ are not small enough against the R^2-term. In fact, the possibility of destabilizing the Starobinsky inflationary scenario by the terms with higher powers of the scalar curvature, in the context of $f(R)$ gravity, was noticed earlier in refs. [60, 61]. The most general Ansatz (85), which is merely *quadratic* in the supercurvature, does not help for that purpose either.

For example, the full $f(R)$-gravity function $f_-(R)$ in eq. (98), which we derived from our $\mathcal{R}^2$-supergravity, gives rise to the inflaton scalar potential

$$V(y) = V_0 \, (11e^y + 3) \left(e^{-y} - 1\right)^2 \tag{117}$$

where $V_0 = (3^3/2^6) M_{\rm Pl}^4/f_2^2$. The corresponding inflationary parameters

$$\varepsilon(y) = \frac{1}{3} \left[\frac{e^y \, (11 + 11e^{-y} + 6e^{-2y})}{(11e^y + 3)(e^{-y} - 1)} \right]^2 \geq \frac{1}{3} \tag{118}$$

and

$$\eta(y) = \frac{2}{3} \frac{(11e^y + 5e^{-y} + 12e^{-2y})}{(11e^y + 3)(e^{-y} - 1)^2} \geq \frac{2}{3} \tag{119}$$

are not small enough for matching the WMAP observational data. A solution to this problem is given in the next Sec. 10.

10 Chaotic inflation in $F(\mathcal{R})$ Supergravity

Let us take now one more step further and consider a new Ansatz for $F(\mathcal{R})$ function in the *cubic* form

$$F(\mathcal{R}) = -\tfrac{1}{2} f_1 \mathcal{R} + \tfrac{1}{2} f_2 \mathcal{R}^2 - \tfrac{1}{6} f_3 \mathcal{R}^3 \tag{120}$$

whose real (positive) coupling constants $f_{1,2,3}$ are of (mass) dimension 2, 1 and 0, respectively. Our conditions on the coefficients are

$$f_3 \gg 1 \; , \qquad f_2^2 \gg f_1 \tag{121}$$

The first condition is needed to have inflation at the curvatures much less than $M^2_{\rm Pl}$ (and to meet observations), while the second condition is needed to have the scalaron (inflaton) mass be much less than $M_{\rm Pl}$, in order to avoid large (gravitational) quantum loop corrections after the end of inflation up to the present time.

The bosonic action is given by eq. (73). In the case of a real scalaron it reduces to

$$L/\sqrt{-g} = 2F'\left[\frac{1}{3}R + 4X^2\right] + 6XF \tag{122}$$

so that the real auxiliary field is a solution to the algebraic equation

$$3F + 11F'X + F''\left[\frac{1}{3}R + 4X^2\right] = 0 \tag{123}$$

Stability of the bosonic embedding in supergravity requires $F'(X) < 0$ (Sec. 9). In the case (120) it gives rise to the condition $f_2^2 < f_1 f_3$. For simplicity here, we will assume a stronger condition,

$$f_2^2 \ll f_1 f_3 \tag{124}$$

Then the second term on the right-hand-side of eq. (120) will not affect inflation, as is shown below.

Equation (122) with the Ansatz (120) reads

$$L = -5f_3X^4 + 11f_2X^3 - (7f_1 + \tfrac{1}{3}f_3R)X^2 + \tfrac{2}{3}f_2RX - \tfrac{1}{3}f_1R \tag{125}$$

and gives rise to a cubic equation on X,

$$X^3 - \left(\frac{33f_2}{20f_3}\right)X^2 + \left(\frac{7f_1}{10f_3} + \frac{1}{30}R\right)X - \frac{f_2}{30f_3}R = 0 \tag{126}$$

We find three consecutive (overlapping) regimes.

- The high curvature regime including inflation is given by

$$\delta R < 0 \quad \text{and} \quad \frac{|\delta R|}{R_0} \gg \left(\frac{f_2^2}{f_1 f_3}\right)^{1/3} \tag{127}$$

  where we have introduced the notation $R_0 = 21f_1/f_3 > 0$ and $\delta R = R + R_0$. With our sign conventions we have $R < 0$ during the de Sitter and matter dominated stages. In the regime (127) the f_2-dependent terms in eqs. (125) and (126) can be neglected, and we get

$$X^2 = -\tfrac{1}{30}\delta R \tag{128}$$

  and

$$L = -\frac{f_1}{3}R + \frac{f_3}{180}(R + R_0)^2 \tag{129}$$

It closely reproduces the Starobinsly inflationary model (Sec. 2) since inflation occurs at $|R| \gg R_0$. In particular, we can identify

$$f_3 = \frac{15M_{\rm Pl}^2}{M_{\rm inf}^2} \tag{130}$$

It is worth mentioning that we cannot simply set $f_2 = 0$ in eq. (120) because it would imply $X = 0$ and $L = -\frac{f_1}{3}R$ for $\delta R > 0$. As a result of that the scalar degree of freedom would disappear that would lead to the breaking of a regular Cauchy evolution. Therefore, the second term in eq. (120) is needed to remove that degeneracy.

- The intermediate (post-inflationary) regime is given by

$$\frac{|\delta R|}{R_0} \ll 1 \tag{131}$$

In this case X is given by a root of the cubic equation

$$30X^3 + (\delta R)X + \frac{f_2 R_0}{f_3} = 0 \tag{132}$$

It also implies that the 2nd term in eq. (126) is always small. Equation (132) reduces to eq. (128) under the conditions (127).

- The low-curvature regime (up to $R = 0$) is given by

$$\delta R > 0 \quad \text{and} \quad \frac{\delta R}{R_0} \gg \left(\frac{f_2^2}{f_1 f_3}\right)^{1/3} \tag{133}$$

It yields

$$X = \frac{f_2 R}{f_3(R + R_0)} \tag{134}$$

and

$$L = -\frac{f_1}{3}R + \frac{f_2^2 R^2}{3f_3(R + R_0)} \tag{135}$$

It is now clear that f_1 should be equal to $3M_{Pl}^2/2$ in order to obtain the correctly normalized Einstein gravity at $|R| \ll R_0$. In this regime the scalaron mass squared is given by

$$\frac{1}{3\,|f''(R)|} = \frac{f_3 R_0 M_{\rm Pl}^2}{4f_2^2} = \frac{21 f_1}{4f_2^2} M_{\rm Pl}^2 = \frac{63 M_{\rm Pl}^4}{8f_2^2} \tag{136}$$

in agreement with the case of the absence of the $\mathcal{R}^3$ term, studied in the previous section. The scalaron mass squared (136) is much less than M_{Pl}^2 indeed, due to the second inequality in eq. (121), but it is much more than one at the end of inflation $(\sim M^2)$.

It is worth noticing that the corrections to the Einstein action in eqs. (129) and (135) are of ther same order (and small) at the borders of the intermediate region (131).

The roots of the cubic equation (126) are given by the textbook (Cardano) formula [62], though that formula is not very illuminating in a generic case. The Cardano formula greatly simplifies in the most interesting (high curvature) regime where inflation takes place, and the Cardano discriminant is

$$D \approx \left(\tfrac{R}{90}\right)^3 < 0 \tag{137}$$

It implies that all three roots are real and unequal. The Cardano formula yields the roots

$$X_{1,2,3} \approx \frac{2}{3}\sqrt{\frac{-R}{10}}\cos\left(\frac{27}{4f_3\sqrt{-10R/f_2^2}} + C_{1,2,3}\right) + \frac{11f_2}{20f_3} \tag{138}$$

where the constant $C_{1,2,3}$ takes the values $(\pi/6, 5\pi/6, 3\pi/2)$.

As regards the leading terms, eqs. (125) and (138) result in the $(-R)^{3/2}$ correction to the $(R+R^2)$-terms in the effective Lagrangian in the high-curvature regime $|R| \gg f_2^2/f_3^2$. In order to verify that this correction does not change our results under the conditions (127), let us consider the $f(R)$-gravity model with

$$\tilde{f}(R) = R - b(-R)^{3/2} - aR^2 \tag{139}$$

whose parameters $a > 0$ and $b > 0$ are subject to the conditions $a \gg 1$ and $b/a^2 \ll 1$. It is easy to check that $\tilde{f}'(R) > 0$ for $R \in (-\infty, 0]$, as is needed for (classical) stability.

Any $f(R)$ gravity model is classically equivalent to the scalar-tensor gravity with certain scalar potential (Sec. 3). The scalar potential can be calculated from a given function $f(R)$ along the standard lines (Sec. 3). We find (in the high curvature regime)

$$V(y) = \frac{1}{8a}\left(1-e^{-y}\right)^2 + \frac{b}{8\sqrt{2a}}e^{-2y}\left(e^y - 1\right)^{3/2} \tag{140}$$

in terms of the inflaton field y. The first term of this equation is the scalar potential associated with the pure $(R+R^2)$ model, and the 2nd term is the correction due to the $R^{3/2}$-term in eq. (139). It is now clear that for large positive y the vacuum energy in the first term dominates and drives inflation until the vacuum energy is compensated by the y-dependent terms near $e^y = 1$.

It can be verified along the lines of ref. [35] that the formula for scalar perturbations remains the same as that for the model (7), ie. $\Delta_{\mathcal{R}}^2 \approx N^2 M_{\rm inf}^2/(24\pi^2 M_{\rm Pl}^2)$, where N is the number of e-folds from the end of inflation. So, to fit the observational data, one has to choose

$$f_3 \approx 5N_e^2/(8\pi^2\Delta_{\mathcal{R}}^2) \approx 6.5\cdot 10^{10}(N_e/50)^2 \tag{141}$$

Here the value of $\Delta_{\mathcal{R}}$ is taken from ref. [12] and the subscript $\mathcal{R}$ has a different meaning from the rest of this review.

We conclude that the model (120) with a sufficiently small f_2 obeying the conditions (121) and (124) gives a viable realization of the chaotic $(R + R^2)$-type inflation in supergravity. The only significant difference with respect to the original $(R + R^2)$ inflationary model is the scalaron mass that becomes much larger than M in supergravity, soon after the end of inflation when δR becomes positive. However, it only makes the scalaron decay faster and creation of the usual matter (reheating) more effective.

The whole series in powers of $\mathcal{R}$ may also be considered, instead of the limited Ansatz (120). The only necessary condition for embedding inflation is that f_3 should be anomalously large. When the curvature grows, the $\mathcal{R}^3$-term should become important much earlier than the convergence radius of the whole series without that term. Of course, it means that viable inflation may not occur for any function $F(\mathcal{R})$ but only inside a small region of non-zero measure in the space of all those functions. However, the same is true for all known inflationary models, so the very existence of inflation has to be taken from the observational data, not from a pure thought.

The results of this Section can be considered as the viable alternative to the earlier fundamental proposals [52, 53] for realization of chaotic inflation in supergravity. But inflation is not the only target of our construction. As is well known [21, 22, 63], the scalaron decays into pairs of particles and anti-particles of quantum matter fields, while its decay into gravitons is strongly suppressed [64]. It thus represents the *universal mechanism of viable reheating* after inflation and provides a transition to the subsequent hot radiation-dominated stage of the Universe evolution. In its turn, it leads to the standard primordial nucleosynthesis (BBN) after. In $F(R)$ supergravity the scalaron has a pseudo-scalar superpartner (axion) that may be the source of a strong CP-violation and then, subsequently, lepto- and baryo-genesis that naturally lead to baryon (matter-antimatter) asymmetry [68, 69] — see Secs. 13 and 15 for more.

11 Cosmological Constant in $F(\mathcal{R})$ Supergravity

The Standard (Λ-CDM) Model in cosmology gives a phenomenological description of the observed *Dark Energy* (DE) and *Dark Matter* (DM). It is based on the use of a small positive cosmological constant Λ and a *Cold Dark Matter* (CDM), and is consistent with all observations coming from the existing cosmological, Solar system and ground-based laboratory data. However, the Λ-CDM Model cannot be the ultimate answer to DE, since it implies its time-independence. For example, the 'primordial' DE responsible for inflation in the early Universe was different from Λ and unstable. The *dynamical* (ie. time-dependent) models of DE can be easily constructed by using the $f(R)$ gravity theories, defined via replacing the scalar curvature R by a function $f(R)$ in the gravitational action. The $f(R)$ gravity provides the self-consistent non-trivial alternative to the Λ-CDM Model. Viable $f(R)$-gravity-based models of the current DE are also known [70, 71, 72], and the combined inflationary-DE models are possible too [27].

The natural question arises, whether $F(\mathcal{R})$ supergravity is also capable to describe

the present DE and eg., a *positive* cosmological constant. It is non-trivial because the standard (pure) supergravity can only have a zero or negative cosmological constant. In this Section we further extend the Ansatz used in Sec. 10 for the F-function, and apply it to get a positive cosmological constant in the regime of *low* spacetime curvature.

Throughout this Section we use the units $c = \hbar = M_{\rm Pl} = 1$. We recall that an AdS-spacetime has a positive scalar curvature, and a dS-spacetime has a negative scalar curvature in our notation.

The embedding of $f(R)$ gravity into $F(\mathcal{R})$ supergravity is given by (Sec. 8)

$$f(R) = f(R, X(R)) \tag{142}$$

where the function $f(R, X)$ (or the gravity Lagrangian $\mathcal{L}$) is defined by

$$\mathcal{L} = f(R, X) = 2F'(X)\left[\frac{1}{3}R + 4X^2\right] + 6XF(X) \tag{143}$$

and the function $X = X(R)$ is determined by solving an algebraic equation,

$$\frac{\partial f(R, X)}{\partial X} = 0 \tag{144}$$

The cosmological constant in $F(\mathcal{R})$ supergravity is thus given by

$$\Lambda = -f(0, X_0) \tag{145}$$

where $X_0 = X(0)$. It should be mentioned that X_0 represents the vacuum expectation value of the auxiliary field X that determines the scale of the supersymmetry breaking. Both inflation and DE imply $X_0 \neq 0$.

To describe DE in the present Universe, ie. in the regime of *low* spacetime curvature R, the function $f(R)$ should be close to the Einstein-Hilbert (linear) function $f_{\rm EH}(R)$ with a small positive Λ,

$$|f(R) - f_{\rm EH}(R)| \ll |f_{\rm EH}(R)|\,, \quad |f'(R) - f'_{\rm EH}| \ll 1, \quad |Rf''(R)| \ll 1 \tag{146}$$

ie. $f(R) \approx -\frac{1}{2}R - \Lambda$ for small R with the very small and positive $\Lambda \approx 10^{-118}(M^4_{\rm Pl})$.

Equations (143) and (145) imply

$$\Lambda = -8F'(X_0)X_0^2 - 6X_0F(X_0) \tag{147}$$

where X_0 is a solution to the algebraic equation

$$4X_0^2F''(X_0) + 11X_0F'(X_0) + 3F(X_0) = 0 \tag{148}$$

As is clear from eq. (147), to have $\Lambda \neq 0$, one must have $X_0 \neq 0$, ie. a (spontaneous) supersymmetry breaking. However, in order to proceed further, we need a reasonable Ansatz for the F-function.

The simplest opportunity is given by expanding the function $F(\mathcal{R})$ in Taylor series with respect to $\mathcal{R}$. Since the $N = 1$ chiral superfield $\mathcal{R}$ has X as its leading field component (in θ-expansion), one may expect that the Taylor expansion is a good approximation as long as $|X_0| \ll 1(M_{\rm Pl})$. As was demonstrated in Sec. 10, a viable (successful) description of inflation is possible in $F(\mathcal{R})$ supergravity, when keeping the *cubic* term $\mathcal{R}^3$ in the Taylor expansion of the $F(\mathcal{R})$ function. It is, therefore, natural to expand the function F up to the cubic term with respect to $\mathcal{R}$, and use it as our Ansatz here,

$$F(\mathcal{R}) = f_0 - \frac{1}{2}f_1\mathcal{R} + \frac{1}{2}f_2\mathcal{R}^2 - \frac{1}{6}f_3\mathcal{R}^3 \tag{149}$$

with some real coeffieints f_0, f_1, f_2, f_3. The Ansatz (149) differs from the one used in eq. (120) by the presence of the new parameter f_0 only. It is worth emphasizing here that f_0 is *not* a cosmological constant because one still has to eliminate the auxiliary field X. The stability conditions (Sec. 9) imply

$$f_1 > 0 \ , \qquad f_2 > 0 \ , \qquad f_3 > 0 \tag{150}$$

and

$$f_2^2 < f_1 f_3 \tag{151}$$

Inflation requires $f_3 \gg 1$ and $f_2^2 \gg f_1$. [4] As was already found in Sec. 10, in order to meet the WMAP observations, the parameterf_3 should be approximately equal to $6.5 \cdot 10^{10}(N_e/50)^2$. The cosmological constant in the high-curvature regime does not play a significant role and may be ignored there.

In the low curvature regime, in order to recover the Einstein-Hilbert term, one has to fix $f_1 = 3/2$ (Sec. 10). Then the Ansatz (149) leads to the gravitational Lagrangian

$$f(R,X) = -5f_3X^4 + 11f_2X^3 - \frac{1}{3}f_3\left(R + \frac{63}{2f_3}\right)X^2 + \left(6f_0 + \frac{2}{3}f_2R\right)X - \frac{1}{2}R \tag{152}$$

and the auxiliary field equation

$$X^3 - \frac{33f_2}{20f_3}X^2 + \frac{1}{30}\left(R + \frac{63}{2f_3}\right)X - \frac{1}{30f_3}\left(f_2R + 9f_0\right) = 0 \tag{153}$$

whose formal solution is available via the standard Cardano (Viète) formulae [62].

In the low-curvature regime we find a cubic equation for X_0 in the form

$$X_0^3 - \left(\frac{33f_2}{20f_3}\right)X_0^2 + \left(\frac{21}{20f_3}\right)X_0 - \left(\frac{3f_0}{10f_3}\right) = 0 \tag{154}$$

'Linearizing' eq. (154) with respect to X_0 brings the solution $X_0 = 2f_0/7$ whose substitution into the action (152) gives rise to a *negative* cosmological constant, $\Lambda_0 = -6f_0^2/7$. This way we recover the standard supergravity case.

[4] The stronger condition $f_2^2 \ll f_1 f_3$ has been used in Sec. 10 for simplicity.

Equations (152) and (154) allow us to write down the exact eq. (145) for the cosmological constant in the *factorized* form

$$\Lambda(X_0) = -\frac{11f_2}{4}X_0(X_0 - X_-)(X_0 - X_+) \tag{155}$$

where $X_\pm$ are the roots of the quadratic equation $x^2 - \frac{21}{11f_2}x + \frac{18f_0}{11f_2} = 0$, ie.

$$X_\pm = \frac{21}{22f_2}\left[1 \pm \sqrt{1 - \frac{2^3 \cdot 11}{7^2}f_0 f_2}\right] \tag{156}$$

Since $f_0 f_2$ is supposed to be very small, both roots $X_\pm$ are real and positive.

Equation (155) implies that $\Lambda > 0$ when either (I) $X_0 < 0$, or (II) X_0 is inside the interval (X_-, X_+).

By using *Matematica* we were able to numerically confirm the existence of solutions to eq. (154) in the region (I) when $f_0 < 0$, but not in the region (II). So, to this end, we continue with the region (I) only. All real roots of eq. (154) are given by

$$\begin{aligned} (X_0)_1 = &\; 2\sqrt{-Q}\cos\left(\frac{\vartheta}{3}\right) + \frac{11f_2}{20f_3} \ , \\ (X_0)_2 = &\; 2\sqrt{-Q}\cos\left(\frac{\vartheta+2\pi}{3}\right) + \frac{11f_2}{20f_3} \ , \\ (X_0)_3 = &\; 2\sqrt{-Q}\cos\left(\frac{\vartheta+4\pi}{3}\right) + \frac{11f_2}{20f_3} \ , \end{aligned} \tag{157}$$

in terms of the Cardano-Viète parameters

$$\begin{aligned} Q = &\; -\frac{11f_2}{2^2 \cdot 5f_3} - \frac{7^2}{2^4 \cdot 5^2 f_3^2} \approx -\frac{11f_2}{20f_3} \ , \\ \hat{R} = &\; -\frac{3 \cdot 7 \cdot 11 f_2}{2^5 \cdot 5^2 f_3^2} + \frac{3f_0}{2^2 \cdot 5f_3} + \frac{11^3 f_2^3}{2^6 \cdot 5^3 f_3^3} \approx -\frac{1}{20f_3}\left(-\frac{21}{2}Q + 3f_0\right) \end{aligned} \tag{158}$$

and the angle ϑ defined by

$$\cos\vartheta = \frac{\hat{R}}{\sqrt{-Q^3}} \tag{159}$$

The Cardano discriminant reads $D = \hat{R}^2 + Q^3$. All three roots are real provided that $D < 0$. It is known to be the case in the high-curvature regime (Sec. 10), and it is also the case when f_0 is extremely small. Under our requirements on the parameters the angle ϑ is very close to zero, so the relevant solutions $X_0 < 0$ are given by the 2nd and 3rd lines of eq. (157), with $X_0 \approx f_0/10$.

We thus showed that it is possible to have a *positive* cosmological constant (at low spacetime curvature) in the particular $F(\mathcal{R})$ supergravity (without its coupling to

super-matter) described by the Ansatz (149). The same Ansatz is applicable for describing a viable chaotic inflation in supergravity (at high spacetime curvature). The positive cosmological constant was achieved as the *non-linear* effect (with respect to the superspace curvature and spacetime curvature) in the relatively narrow part of the parameter space (it is, therefore, highly constrained). It also implies the apparent violation of the Strong Energy Condition in our model.

Of course, describing the DE in the present Universe requires enormous fine-tuning of our parameters in the F-function. However, it is the common feature of all known approaches to the DE. Our analysis does not contribute to 'explaining' the smallness of the cosmological constant.

12 Nonminimal Scalar-Curvature Coupling in Gravity and Supergravity, and Higgs inflation

One may pursue different strategies in a theoretical search for inflaton. For instance, inflaton may be either a new exotic particle or something that we already know 'just around the corner'. In this review I advocate the second ('economical') approach. Perhaps, the most 'economical' possibilities are given by Higgs inflation [73, 74, 76] and Starobinsky $(R + R^2)$ inflation (Sec. 2).

According to the cosmology textbooks, a Higgs particle of the Standard Model *cannot* serve as inflaton because the SM parameters are $\lambda \approx 1$, $m_{\rm H} \approx 10^2\ GeV$, and $(\delta T/T) \approx 1$, whereas inflation requires (see Sec. 4) $\lambda \approx 10^{-13}$, $m_{\rm inf} \approx 10^{13}\ GeV$, and $(\delta T/T) \approx 10^{-5}$. Nevertheless, it is possible to reach the required values when assuming that Higgs particle is *nonminimally* coupled to gravity [73, 74, 76]. For instance, adding the nonminimal coupling of the Higgs field to the scalar spacetime curvature is natural in curved spacetime because it is required by renormalization [77]. It is worth recalling here that in the Starobinsky inflaton [21, 22] the inflaton field is the spin-0 part of spacetime metric.

In this Section we compare the inflationary scalar potential, derived by the use of the nonminimal coupling [73, 74, 76], with the scalar potential that follows from the $(R + R^2)$ inflationary model (Sec. 2), and confirm that they are *the same*. Then we also upgrade that equivalence to supergravity. In this section we set $M_{\rm Pl} = 1$ too.

The original motivation of refs. [73, 74, 76] is based on the assumption that there is no new physics beyond the Standard Model up to the Planck scale. Then it is natural to search for the most economical mechanism of inflation by identifying inflaton with Higgs particle. Later, in this Section, we assume that there is the new physics beyond the Standard Model, and it is given by s upersymmetry. Then it is quite natural to search for the most economical mechanism of inflation in the context of supergravity. Moreover, we do not have to identify our inflaton with a Higgs particle of the Minimal Supersymmetric Standard Model.

Let us begin with the 4D Lagrangian

$$\mathcal{L}_{\rm J} = \sqrt{-g_{\rm J}} \left\{ -\frac{1}{2}(1+\xi\phi_{\rm J}^2)R_{\rm J} + \frac{1}{2}g_{\rm J}^{\mu\nu}\partial_\mu\phi_{\rm J}\partial_\nu\phi_{\rm J} - V(\phi_{\rm J}) \right\} \tag{160}$$

where we have introduced the real scalar field $\phi_{\rm J}(x)$, nonminimally coupled to gravity (with the coupling constant ξ) in Jordan frame, with the Higgs-like scalar potential

$$V(\phi_{\rm J}) = \frac{\lambda}{4}(\phi_{\rm J}^2 - v^2)^2 \tag{161}$$

The action (160) can be rewritten to Einstein frame by redefining the metric via a Weyl transformation,

$$g^{\mu\nu} = \frac{g_{\rm J}^{\mu\nu}}{(1+\xi\phi_{\rm J}^2)} \tag{162}$$

It gives rise to the standard Einstein-Hilbert term $(-\frac{1}{2}R)$ for gravity in the Lagrangian. However, it also leads to a nonminimal (or noncanonical) kinetic term of the scalar field $\phi_{\rm J}$. To get the canonical kinetic term, a scalar field redefinition is needed, $\phi_{\rm J} \to \varphi(\phi_{\rm J})$, subject to the condition

$$\frac{d\varphi}{d\phi_{\rm J}} = \frac{\sqrt{1+\xi(1+6\xi)\phi_{\rm J}^2}}{1+\xi\phi_{\rm J}^2} \tag{163}$$

As a result, the non-minimal theory (160) is classically equivalent to the standard (canonical) theory of the scalar field $\varphi(x)$ minimally coupled to gravity,

$$\mathcal{L}_{\rm E} = \sqrt{-g} \left\{ -\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi) \right\} \tag{164}$$

with the scalar potential

$$V(\varphi) = \frac{V(\phi_{\rm J}(\varphi))}{[1+\xi\phi_{\rm J}^2(\varphi)]^2} \tag{165}$$

Given a large positive $\xi \gg 1$, in the small field limit one finds from eq. (163) that $\phi_{\rm J} \approx \varphi$, whereas in the large φ l imit one gets

$$\varphi \approx \sqrt{\frac{3}{2}}\log(1+\xi\phi_{\rm J}^2) \tag{166}$$

Then eq. (165) yields the scalar potential:
(i) in the *very small* field limit, $\varphi < \sqrt{\frac{2}{3}}\xi^{-1}$, as

$$V_{\rm vs}(\varphi) \approx \frac{\lambda}{4}\varphi^4 \tag{167}$$

(ii) in the *small* field limit, $\sqrt{\frac{2}{3}}\xi^{-1} < \varphi \ll \sqrt{\frac{3}{2}}$, as

$$V_{\rm s}(\varphi) \approx \frac{\lambda}{6\xi^2}\varphi^2, \tag{168}$$

(iii) and in the *large* field limit, $\varphi \gg \sqrt{\frac{2}{3}}\xi^{-1}$, as

$$V(\varphi) \approx \frac{\lambda}{4\xi^2}\left(1 - \exp\left[-\sqrt{\frac{2}{3}}\varphi\right]\right)^2 \tag{169}$$

We have assumed here that $\xi \gg 1$ and $v\xi \ll 1$.

Identifying inflaton with Higgs particle requires the parameter v to be of the order of weak scale, and the coupling λ to be the Higgs boson selfcoupling at the inflationary scale. The scalar potential (169) is perfectly suitable to support a slow-roll inflation, while its consistency with the WMAP normalization condition (Sec. 4) for the observed CMB amplitude of density perturbations at the e-foldings number $N_e = 55$ gives rise to the relation $\xi/\sqrt{\lambda} \approx 5 \cdot 10^4$ [73, 74, 76].

The scalar potential (168) corresponds to the post-inflationary matter-dominated epoch described by the oscillating inflaton field φ with the frequency

$$\omega = \sqrt{\frac{\lambda}{3}}\,\xi^{-1} = M_{\rm inf} \tag{170}$$

When gravity is extended to 4D, $N = 1$ supergravity, any physical real scalar field should be complexified, becoming the leading complex scalar field component of a chiral (scalar) matter supermultiplet. In a curved superspace of $N = 1$ supergravity, the chiral matter supermultiplet is described by a covariantly chiral superfield Φ obeying the constaraint $\overline{\nabla}_{\dot{\alpha}}\Phi = 0$. The standard (generic and minimally coupled) matter-supergravity action is given by in superspace by eqs. (45) and and (47), namely,

$$S_{\rm MSG} = -3\int d^4xd^4\theta E^{-1}\exp\left[-\frac{1}{3}K(\Phi,\overline{\Phi})\right] + \left\{\int d^4xd^2\theta\mathcal{E}W(\Phi) + {\rm H.c.}\right\} \tag{171}$$

in terms of the Kähler potential $K = -3\log(-\frac{1}{3}\Omega)$ and the superpotential W of the chiral supermatter, and the full density E and the chiral density $\mathcal{E}$ of the superspace supergravity (Sec. 5).

The non-minimal matter-supergravity coupling in superspace reads

$$S_{\rm NM} = \int d^4xd^2\theta\mathcal{E}X(\Phi)\mathcal{R} + {\rm H.c.} \tag{172}$$

in terms of the chiral function $X(\Phi)$ and the N=1 chiral scalar supercurvature superfield $\mathcal{R}$ obeying $\overline{\nabla}_{\dot{\alpha}}\mathcal{R} = 0$. In terms of the field components of the superfields the non-minimal action (172) is given by

$$\int d^4xd^2\theta\mathcal{E}X(\Phi)\mathcal{R} + {\rm H.c.} = -\frac{1}{6}\int d^4x\sqrt{-g}X(\phi_c)R + {\rm H.c.} + \ldots \tag{173}$$

stand for the fermionic terms, and $\phi_c = \Phi| = \phi + i\chi$ is the leading complex scalar field component of the superfield Φ. Given $X(\Phi) = -\xi\Phi^2$ with the real coupling constant ξ, we find the bosonic contribution

$$S_{\rm NM,bos.} = \frac{1}{6}\xi\int d^4x\sqrt{-g}\left(\phi^2 - \chi^2\right)R \tag{174}$$

It is worth noticing that the supersymmetrizable (bosonic) non-minimal coupling reads $\left[\phi_c^2 + (\phi_c^\dagger)^2\right] R$, not $(\phi_c^\dagger \phi_c)R$.

Let us now introduce the manifestly supersymmetric nonminimal action (in Jordan frame) as

$$S = S_{\rm MSG} + S_{\rm NM} \tag{175}$$

In curved superspace of $N = 1$ supergravity the (Siegel's) chiral integration rule

$$\int d^4x d^2\theta \mathcal{E} \mathcal{L}_{\rm ch} = \int d^4x d^4\theta E^{-1} \frac{\mathcal{L}_{\rm ch}}{\mathcal{R}} \tag{176}$$

applies to any chiral superfield Lagrangian $\mathcal{L}_{\rm ch}$ with $\overline{\nabla}_{\dot{\alpha}} \mathcal{L}_{\rm ch} = 0$. It is, therefore, possible to rewrite eq. (172) to the equivalent form

$$S_{\rm NM} = \int d^4x d^4\theta E^{-1} \left[X(\Phi) + \overline{X}(\overline{\Phi})\right] \tag{177}$$

We conclude that adding $S_{\rm NM}$ to $S_{\rm MSG}$ is equivalent to the simple change of the Ω-potential as (*cf.* ref. [78])

$$\Omega \to \Omega_{\rm NM} = \Omega + X(\Phi) + \bar{X}(\overline{\Phi}) \tag{178}$$

It amounts to the change of the Kähler potential as

$$K_{\rm NM} = -3 \ln \left[e^{-K/3} - \frac{X(\Phi) + \overline{X}(\overline{\Phi})}{3} \right] \tag{179}$$

The scalar potential in the matter-coupled supergravity (171) is given by eq. (57),

$$V(\phi, \bar{\phi}) = e^G \left[\left(\frac{\partial^2 G}{\partial \phi \partial \bar{\phi}} \right)^{-1} \frac{\partial G}{\partial \phi} \frac{\partial G}{\partial \bar{\phi}} - 3 \right] \tag{180}$$

in terms of the Kähler-gauge-invariant function (53), ie.

$$G = K + \ln |W|^2 \tag{181}$$

Hence, in the nonminimal case (175) we have

$$G_{\rm NM} = K_{\rm NM} + \ln |W|^2 \tag{182}$$

Contrary to the bosonic case, one gets a nontrivial Kähler potential $K_{\rm NM}$, ie. a *Non-Linear Sigma-Model* (NLSM) as the kinetic term of $\phi_c = \phi + i\chi$ (see ref. [51] for more about the NLSM). Since the NLSM target space in general has a nonvanishing curvature, no field redefinition generically exist that could bring the kinetic term to the free (canonical) form with its Kähler potential $K_{\rm free} = \overline{\Phi}\Phi$.

Let's now consider the full action (175) under the slow-roll condition, ie. when the contribution of the kinetic term is negligible. Then eq. (175) takes the truly chiral form

$$S_{\text{ch.}} = \int d^4x d^2\theta \mathcal{E} \left[X(\Phi)\mathcal{R} + W(\Phi) \right] + \text{H.c.} \tag{183}$$

When choosing X as the independent chiral superfield, $S_{\text{ch.}}$ can be rewritten to the form

$$S_{\text{ch.}} = \int d^4x d^2\theta \mathcal{E} \left[X\mathcal{R} - \mathcal{Z}(X) \right] + \text{H.c.} \tag{184}$$

where we have introduced the notation

$$\mathcal{Z}(X) = -W(\Phi(X)) \tag{185}$$

In its turn, the action (184) is equivalent to the chiral $F(\mathcal{R})$ supergravity action (35), whose function F is related to the function $\mathcal{Z}$ via Legendre transformation (Sec. 6)

$$\mathcal{Z} = X\mathcal{R} - F \ , \quad F'(\mathcal{R}) = X \quad \text{and} \quad \mathcal{Z}'(X) = \mathcal{R} \tag{186}$$

It implies the equivalence between the reduced action (183) and the corresponding $F(\mathcal{R})$ supergravity whose F-finction obeys eq. (186).

Next, let us consider the special case of eq. (183) when the superpotential is given by

$$W(\Phi) = \frac{1}{2}m\Phi^2 + \frac{1}{6}\tilde{\lambda}\Phi^3 \tag{187}$$

with the real coupling constants $m > 0$ and $\tilde{\lambda} > 0$. The model (187) is known as the *Wess-Zumino* (WZ) model in 4D, $N = 1$ rigid supersymmetry. It has the most general renormalizable scalar superpotential in the absence of supergravity. In terms of the field components, it gives rise to the Higgs-like scalar potential.

For simplicity, let us take a cubic superpotential,

$$W_3(\Phi) = \frac{1}{6}\tilde{\lambda}\Phi^3 \tag{188}$$

or just assume that this term dominates in the superpotential (187), and choose the $X(\Phi)$-function in eq. (183) in the form

$$X(\Phi) = -\xi\Phi^2 \tag{189}$$

with a large positive coefficient ξ, $\xi > 0$ and $\xi \gg 1$, in accordance with eqs. (173) and (174).

Let us also simplify the F-function of eq. (120) by keeping only the most relevant cubic term,

$$F_3(\mathcal{R}) = -\frac{1}{6}f_3\mathcal{R}^3 \tag{190}$$

It is straightforward to calculate the $\mathcal{Z}$-function for the F-function (190) by using eq. (186). We find

$$-X = \frac{1}{2} f_3 \mathcal{R}^2 \quad \text{and} \quad \mathcal{Z}'(X) = \sqrt{\frac{-2X}{f_3}} \tag{191}$$

Integrating the last equation with respect to X yields

$$\mathcal{Z}(X) = -\frac{2}{3}\sqrt{\frac{2}{f_3}}(-X)^{3/2} = -\frac{2\sqrt{2}}{3}\frac{\xi^{3/2}}{f_3^{1/2}}\Phi^3 \tag{192}$$

where we have used eq. (189). In accordance to eq. (185), the $F(\mathcal{R})$-supergravity $\mathcal{Z}$-potential (192) implies the superpotential

$$W_{\rm KS}(\Phi) = \frac{2\sqrt{2}}{3}\frac{\xi^{3/2}}{f_3^{1/2}}\Phi^3 \tag{193}$$

It coincides with the superpotential (188) of the WZ-model, provided that we identify the couplings as

$$f_3 = \frac{32\xi^3}{\tilde{\lambda}^2} \tag{194}$$

We conclude that the original nonminimally coupled matter-supergravity theory (175) in the slow-roll approximation with the superpotential (188) is classically equivalent to the $F(\mathcal{R})$-supergravity theory with the F-function given by eq. (190) when the couplings are related by eq. (194).

The inflaton mass M in the supersymmertic case, according to eqs. (130) and (194), is given by

$$M^2_{\rm inf} = \frac{15\tilde{\lambda}^2}{32\xi^3} \tag{195}$$

Since the value of $M_{\rm inf}$ is fixed by the WMAP normalization (Sec. 4), the value of ξ in the supersymmetric case is $\xi^3_{\rm susy} = (45/32)\xi^2_{\rm bos}$, or $\xi_{\rm susy} \approx 10^3$, ie. is *lower* than that in the bosonic case. We have asssumed here that $\tilde{\lambda} \approx \mathcal{O}(1)$.

The established equivalence begs for a fundamental reason. In the high-curvature (inflationary) regime the R^2-term dominates over the R-term in the Starobinsky action (7), while the coupling constant in front of the R^2-action is dimensionless (Sect. 2). The Higgs inflation is based on the Lagrangian (160) with the relevant scalar potential $V_4 = \frac{1}{4}\lambda\phi^4_J$ (the parameter v is irrelevant for inflation), whose coupling constants ξ and λ are also dimensionless. Therefore, both relevant actions are *scale invariant*. Inflation breaks that symmetry spontaneously.

The supersymmetric case is similar: the nonminimal action (183) with the X-function (189) and the superpotential (188) also have only dimensionless coupling constants ξ and $\tilde{\lambda}$, while the same it true for the $F(\mathcal{R})$-supergravity action with the

F-function (190), whose coupling constant f_3 is dimensionless too. Therefore, those actions are both *scale invariant*, while inflation spontaneously breaks that invariance.

A spontaneous breaking of the scale invariance *necessarily* leads to a Goldstone particle (or dilaton) associated with spontaneously broken dilatations. So, perhaps, Starobinsky scalaron (inflaton) may be identified with the Goldstone dilaton!

The basic field theory model, describing both inflation *and* the subsequent reheating, reads (see eg., eq. (6) in ref. [79])

$$\begin{aligned} L/\sqrt{-g} = & \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}\tilde{\xi}R\chi^2 + \bar{\psi}(i\gamma^\mu\partial_\mu - m_\psi)\psi \\ & - \frac{1}{2}g^2\phi^2\chi^2 - h(\bar{\psi}\psi)\phi \end{aligned} \tag{196}$$

with the inflaton scalar field ϕ interacting with another scalar field χ and a spinor field ψ. The nonminimal supergravity theory (175) with the Wess-Zumino superpotential (187) can be considered as the $N=1$ locally supersymetric extension of the basic model (196) after rescaling ϕ_c to $(1/\sqrt{2})\phi_c$ and identifying $\tilde{\xi} = -\frac{1}{3}\xi$ because of eq. (174). Therefore, *pre-heating* (ie. the nonperturbative enhancement of particle production due to parametric resonance [79]) is a generic feature of supergravity models.

The axion χ and fermion ψ are both requred by supersymmetry, being in the same chiral supermultiplet with the inflaton ϕ. The scalar interactions are

$$V_{\rm int}(\phi,\chi) = m\hat{\lambda}\phi(\phi^2+\chi^2) + \frac{\hat{\lambda}^2}{4}(\phi^2+\chi^2)^2 \tag{197}$$

whereas the Yukawa couplings are given by

$$L_{\rm Yu} = \frac{1}{2}\hat{\lambda}\phi(\bar{\psi}\psi) + \frac{1}{2}\hat{\lambda}\chi(\bar{\psi}i\gamma_5\psi) \tag{198}$$

Supersymmetry implies the unification of couplings since $h = -\frac{1}{2}\hat{\lambda}$ and $g^2 = \hat{\lambda}^2$ in terms of the single coupling constant $\hat{\lambda}$. If supersymmetry is unbroken, the masses of ϕ, χ and ψ are all the same. However, inflation already breaks supersymmetry, so the spontaneously broken supersymmetry is appropriate here.

To conclude, inflationary slow-roll dynamics in Einstein gravity theory with a nonminimal scalar-curvature coupling can be equivalent to that in the certain $f(R)$ gravity theory. We just extended that correspondence to $N=1$ supergravity. The nonminimal coupling in supergravity can be rewritten in terms of the standard ('minimal') $N=1$ matter-coupled supergravity, by using their manifestly supersymmetric formulations in curved superspace. The equivalence relation between the supergravity theory with the nonminimal scalar-curvature coupling and the $F(\mathcal{R})$ supergravity during slow-roll inflation is, therefore, established.

The equivalence is expected to hold even after inflation, during *initial* reheating with harmonic oscillations. In the bosonic case the equivalence holds until the inflaton

field value is higher than $\omega \approx M_{\rm Pl}/\xi_{\rm bos} \approx 10^{-5} M_{\rm Pl}$. In the superymmetric case we have the same bound $\omega \approx M_{\rm Pl}/\xi^{3/2}_{\rm susy} \approx 10^{-5} M_{\rm Pl}$.

The Higgs inflation and the renormalization group can be used to compute the mass of a Higgs particle in the Standard Model by descending from the inflationary scale to the electro-weak scale. For example, in the *two-loop* approximation one finds [80]

$$129\, GeV < m_H < 194\, GeV \tag{199}$$

with the theoretical uncertainty of about $\pm 2\, GeV$. It is to be compared to the observed bounds known from direct searchers of Higgs particle. For instance, the most recent (preliminary) data from the LHC in 2011 implies

$$\text{LHC (ATLAS, CMS)}: \ 124\, GeV < m_H < 135\, GeV \tag{200}$$

Therefore, the Higgs inflation is possible in the *upper half* of the *experimentally*-allowed region (200), though in *the lowest* portion of the *theoretically*-allowed region (199). It is worth noticing here that in a *supersymmetric* extension of the SM (like the MSSM and NMSSM) there are more particles, when compared to the bosonic SM. Hence, the SUSY renormalization group trajectory is going to be steeper, while the theoretical SUSY bounds on the Higgs mass at the electro-weak scale are going to be *lower* than those in eq. (199).

13 Reheating and Quantum Particle Production

Reheating is a transfer of energy from inflaton to ordinary particles and fields.

The classical solution (neglecting particle production) near the minimum of the inflaton scalar potential reads

$$a(t) \approx a_0 \left(\frac{t}{t_0}\right)^{2/3} \qquad \text{and} \qquad \varphi(t) \approx \left(\frac{M_{\rm Pl}}{3M_{\rm inf}}\right) \frac{\cos[M_{\rm inf}(t-t_0)]}{t-t_0} \tag{201}$$

A *time-dependent* classical spacetime background leads to *quantum* production of particles with masses $m < \omega = M_{\rm inf}$ [77]. Actually, the amplitude of φ-oscillations decreases much faster [79], namely, as

$$\exp[-\frac{1}{2}(3H + \Gamma)t] \tag{202}$$

via inflaton decay and the universe expansion, as the solution to the inflaton equation

$$\ddot{\varphi} + 3H\, \dot{\varphi} + (m^2 + \Pi)\varphi = 0 \tag{203}$$

Here Π denotes the polarization operator that effectively describes particle production. Unitarity (optical theorem) requires $\mathrm{Im}(\Pi) = m\Gamma$. The assumption $m \gg H$ has also been used here [79].

The Starobinsky model in Jordan frame,

$$S = \int d^4x \sqrt{-g_J} f_S(R_J) + S_{\rm SM}(g^{\mu\nu}{}_J, \psi) \tag{204}$$

after the conformal transformation to Einstein frame reads

$$S = S_{\rm scalar-tensor\ gravity}(g_{\mu\nu}, \varphi) + S_{\rm SM}(g^{\mu\nu} e^{-\sigma\varphi}, \psi) \tag{205}$$

so that the inflaton φ couples to all non-conformal terms and fields ψ, due to the universality of gravitational interaction. Therefore, the Starobinsky inflation also has the *universal* mechanism of particle production.

The perturbative decay rates of the inflaton into a pair of scalars (s) or into a pair of spin-1/2 fermions (f) are given by [21, 22, 84]

$$\Gamma_{\varphi\to ss} = \frac{M^3_{\rm inf}}{192\pi M^2_{\rm Pl}} \qquad \text{and} \qquad \Gamma_{\varphi\to ff} = \frac{M_{\rm inf} M^2_f}{48\pi M^2_{\rm Pl}}\ , \tag{206}$$

respectively. The perturbative decay rate of the inflaton into a pair of *gravitini* is [85]

$$\Gamma_{\varphi\to 2\psi_{3/2}} = \frac{|G_{,\varphi}|^2}{288\pi} \frac{M^5_{\rm inf}}{m^2_{3/2} M^2_{\rm Pl}} \tag{207}$$

Being proportional to $M^5_{\rm inf}$, eq. (207) may lead to the cosmologically disastrous gravitino overproduction [86]. However, when the expectation value of the dilaton is of the order $M_{\rm Pl}$ (it is the case in the Starobinsky inflation), one can demonstrate that eq. (207) reduces to the scalar decay rate (206) proportional to $M^3_{\rm inf}$ [86].

There is *no* parametric resonance enhancement here because the produced particles rapidly scatter. The energy transfers by the time $t_{\rm reh} \geq \left(\sum_{s,f} \Gamma_{s,f}\right)^{-1}$. One finds the *reheating temperature* [36, 87]

$$T_{\rm reh} \propto \sqrt{\frac{M_{\rm Pl}\Gamma}{(\#d.o.f.)^{1/2}}} \approx 10^9 \text{ GeV} \tag{208}$$

that gives the maximal temperature of the primordial plasma.

In the context of supergravity coupled to the supersymmetric matter (like MSSM) gravitino can be either LSP (= the lightest sparticle) or NLSP (= not LSP). In the LSP case (that usually happens with gauge mediation of supersymmetry breaking and $m_{3/2} \ll 10^2$ GeV) gravitino is stable due to the R-parity conservation. If gravitino is NLSP, then it is unstable (it usually happens with gravity- or anomaly- mediation of supersymmetry breaking, and $m_{3/2} \gg 10^2$ GeV). Unstable gravitino can decay into LSP. See ref. [88] for a review of mediation of supersymmetry breaking from the hidden sector to the visible sector.

Stable gravitino may be the dominant part of *Cold Dark Matter* (CDM) [89]. There exist severe Big Bang Nucleosynthesis (BBN) [5] constraints on the overproduction of ^3He in that case, which give rise to the upper bound on the reheating temperature of thermally produced gravitini, $T_{\rm reh} < 10^{5\div 6}$ GeV [75, 91]. The reheating temperature (208) is unrelated to that bound because it corresponds to the much earlier time in the history of the Universe.

When gravitino is NLSP of mass $m_{3/2} \gg 10^2$ GeV, the BBN constraints are drastically relaxed because the gravitino lifetime becomes much shorter than the BBN time (about 1 sec) [75, 91]. In that case the most likely CDM candidate is MSSM neutralino, while the reheating temperature may be as high as 10^{10} GeV [91].

An overproduction of gravitini from inflaton decay and scattering processes should be avoided, in order to prevent overclosure of the universe. The cosmological constraints on gravitino abundances were formulated in ref. [86]. Those constraints are very model-dependent.

The rate of decay changes with time, along with the decreasing amplitude of inflaton oscillations. It stops when the decay rate becomes smaller than the production rate. Then the particle production accelerates (called *pre-heating*, or true BB!) due to the parametric resonance enhancement [79]. The reheating rapidly transfers most of energy to radiation, and leads to a radiation-dominated universe with $a \propto t^{1/2}$.

In the matter-coupled $F(\mathcal{R})$ supergravity with the action

$$S = \left[\int d^4x d^2\theta \, \mathcal{E} F(\mathcal{R}) + \text{H.c.} \right] + S_{\rm SSM}(E, \Psi) \tag{209}$$

after the super-Weyl transformation, $\mathcal{E} \to \mathcal{E} e^{3\Phi}$, we get

$$S = S_{\rm scalar-tensor\ supergravity}(E, \Phi) + S_{\rm SSM}(e^{\Phi + \overline{\Phi}} E, \Psi) \tag{210}$$

so that the superscalaron Φ is *universally* coupled to the SSM matter superfields Ψ.

14 Conclusion

- A manifestly $4D$, $N = 1$ supersymmetric *extension* of $f(R)$ gravity exist, it is *chiral* and is parametrized by a holomorphic function. An $F(\mathcal{R})$ supergravity is classically *equivalent* to the scalar-tensor theory of a chiral scalar superfield (with certain Kähler potential and superpotential) miminally coupled to the $N = 1$ Poincaré supergravity in four spacetime dimensions (with nontrivial G and K), ie. the $N = 1$ supersymmetric quintessence.

The *classical* equivalence between the $F(\mathcal{R})$ supergravity and the quintessence N=1 supergravity has the same physical contents as the classical equivalence between $f(R)$ gravity and scalar-tensor gravity, ie. *the same* inflaton scalar potential and, therefore, the same inflationary dynamics. However, the physical nature of inflaton in the

[5] See ref. [90] for a review of BBN.

$f(R)$ gravity and the scalar-tensor gravity is very different. In the $f(R)$ gravity the inflaton field is the spin-0 part of metric, whereas in the scalar-tensor gravity inflaton is a matter particle. The inflaton interactions with other matter fields are, therefore, different in both theories. It gives rise to the different inflaton decay rates and different reheating, ie. implies different physics in the post-inflationary universe.

Similar remarks apply to the equivalence between Higgs inflation and Starobinsky inflation (Sec. 12). The equivalence does not have to be valid *after* inflation. For example, the *reheating* temperature $T_{\rm reh}$ after the Higgs inflation is about $10^{13}\ GeV$ [73, 74, 76], whereas after the Starobinsky inflation one finds $T_{\rm reh} \approx 10^9\ GeV$ [22] — see also the next Section.

By the use of the renormalization group equations in the Standard Model it is possible to relate the inflationary scale with the electro-weak scale, and thus 'predict' the mass of the Higgs particle. In the two-loop approximation one finds $129\ GeV \leq m_{\rm H} \leq 194\ GeV$ [80]. It is to be compared to the direct searches at *LEP:* $m_H > 114\, GeV$ and *LHC:* $m_H < 145\, GeV$. Therefore, the Higgs inflation is consistent with the known experimental bounds the Higs mass in the upper half of the allowed region, $129\, GeV \leq m_{\rm H} \leq 145\, GeV$ [80]. In a supersymmetric Higgs inflation, the theoretical window for the Higgs mass is *lower*.

- It is expected that the classical equivalence is *broken* in quantum theory because the classical equivalence is achieved via a non-trivial field redefinition (Secs. 3 and 6). When doing that field redefinition in the quantum path integrals defining those quantum theories (unter their unitarity bounds), it gives rise to a non-trivial Jacobian that already implies the *quantum inequivalence*, even before taking into account renormalization. [6]

In the supergravity case, there is one more clear reason for the quantum inequivalence between the $F(\mathcal{R})$ supergravity and the clasically equivalent quintessence supergravity. The Kähler potential of the scalar superfield is described by a *full* superspace integral and, therefore, it receives quantum corrections that can easily spoil classical solutions describing an accelerating universe (those corrections are not under control). It was the reason for introduction of flat directions in the Kähler potential and popular realizations of inflation in supergravity by the use of a chiral scalar superpotential along the flat directions [50, 52, 49]. The $F(\mathcal{R})$ supergravity action is truly chiral, so that the function $F(\mathcal{R})$ is already protected against the quantum perturbative corrections given by full superspace integrals. It is the important part of physical motivation for $F(\mathcal{R})$ supergravity. It also explains why we consider $F(\mathcal{R})$ supergravity as the viable and self-consistent alternative to the Kähler flat directions for realizing slow-roll inflation in supergravity.

- The Starobinsky model of chaotic inflation can be embedded into $F(\mathcal{R})$ supergravity, thus providing the new and truly viable explicit realization of chaotic inflation in supergravity, together with the simple solution to the η-problem, by using just supergravity!

[6] See ref. [81] for the first steps of quantization with a higher time derivative.

- A simple extension of our inflationary model (Sec. 10) has a *positive* cosmological constant in the regime of low spacetime curvature (Sec. 11). It is non-trivial because the standard supergravity with usual matter can only have a negative or vanishing cosmological constant [82]. It happens because the usual (known) matter does not violate the *Strong Energy Condition* (SEC) [83]. A violation of SEC is required for an accelerating universe, and is easily achieved in $f(R)$ gravity due to the fact that the quintessence field in $f(R)$ gravity is part of metric (ie. the unusual matter). Similarly, the quintessence scalar superfield in $F(\mathcal{R})$ supergravity is part of super-vielbein, and also gives rise to a violation of SEC.

In the $F(\mathcal{R})$ supergravity model we considered (Secs. 10 and 11), the effective $f(R)$ gravity function in the high-curvature regime is essentially given by the Starobinsky function $(-\frac{M_{\rm Pl}^2}{2}R+\frac{M_{\rm Pl}^2}{12M_{\rm inf}^2}R^2)$. In the low-curvature regime it is essentially given by the Einstein-Hilbert function with a cosmological constant, $(-\frac{M_{\rm PL}^2}{2}R-\Lambda)$. Therefore, our model has a cosmological solution describing an inflationary universe of the quasi-dS type with $H(t)=(M_{\rm inf}^2/6)(t_{end}-t)$ at early times $t<t_{end}$, and an accelerating universe of the dS-type with $H=\Lambda$ at late times. It is the first time when the unifying description of inflation and Dark Energy is found in the context of supergravity.

The dynamical chiral superfield in $F(\mathcal{R})$ supergravity may be identified with the dilaton-axion chiral superfield in quantum 4D Superstring Theory, when demanding the $SL(2,\mathbf{Z})$ symmetry of the effective action. As is well known, String Theory supports the higher-derivative gravity. In part, the $R^2A(R)$ terms may appear in the bosonic gravitational effective action after superstring compactification. The apparent problem is how to get the anomalously *large* coefficient in front of the $\mathcal{R}^3$-term in the effective $F(\mathcal{R})$ supergravity theory, which would be consistent with string dynamics.

Supersymmetry in $F(R)$ supergravity is already broken by inflation. The anomaly- or gravitationally-mediated supersymmetry breaking may serve as the important element for the new particle phenomenology (beyond the Standard Model) based on the matter-coupled $F(R)$ supergravity theory.

15 Outlook: CP-violation, Baryonic Asymmetry, Lepto- and Baryo-genesis, Non-Gaussianity, Tests

The observed part of our Universe is highly $C-$ and $CP-$asymmetric (no antimatter). Inflation naturally implies a *dynamical* origin of the baryonic matter predominance due to a nonconserved baryon number. The main conditions for the dynamical generation of the cosmological baryon asymmetry in early universe were formulated by A.D. Sakharov in 1967 [65]:

1. nonconservation of baryons (*cf.* SUSY, GUT, EW theory),

2. $C-$ and $CP-$symmetry breaking (confirmed experimentally),

3. deviation from thermal equilibrium in initial hot universe.

The first condition is clearly necessary. And (in theory) there is no fundamental reason for the baryon number conservation. The baryon asymmetry should have originated from spontaneous breaking of the CP symmetry that was present at very early times, so is the need for the second condition. Then the third condition is required by the CPT symmetry, when the CP-violation is compensated by the T-violation, so it has to be no thermal equilibrium.

There exist many scenarios of baryogenesis (see ref. [66] for a review), all designed to explain the observed asymmetry (BBN,CMB):

$$\beta = \frac{n_B - n_{\overline{B}}}{n_\gamma} = (6.0 \pm 0.5) \cdot 10^{-10} \tag{211}$$

Here n_B stands for the concentration of baryons, $n_{\overline{B}}$ for the concentration of anti-baryons, and n_γ for the concentration of photons.

Perhaps, the most popular scenario is the nonthermal *baryo-through-lepto-genesis* [67, 68], ie. a creation of lepton asymmetry by L-nonconserving decays of a heavy ($m \approx 10^{10}$ GeV) Majorana neutrino, and a subsequent transformation of the lepton asymmetry into baryonic asymmetry by CP-symmetric, B-nonconserving and (B-L)-conserving electro-weak processes.

The thermal leptogenesis requires high reheating temperature, $T_{\rm reh} \geq 10^9$ GeV [92], which is consistent with eq. (208).

The matter-coupled $F(\mathcal{R})$ supergravity theory may also contribute towards the *origin* and the *mechanism* of CP-violation and baryon asymmetry, because

- *complex* coefficients of $F(\mathcal{R})$-function and the complex nature of the $F(\mathcal{R})$ supergravity are the simple *source* of explicit CP-violation and complex Yukawa couplings;
- the nonthermal leptogenesis is possible via decay of heavy sterile neutrinos (FY-mechanism) *universally produced* by (super)scalaron decays, or via neutrino oscillations in early universe [93];
- the existence of the natural Cold Dark Matter candidates (*gravitino*, *axion*, *inflatino* or, maybe, *inflaton* itself!) in $F(\mathcal{R})$ supergravity;
- $F(\mathcal{R})$-supergravity can naturally support *hybrid* (or two-field) inflationary models because it already has a pseudo-scalar superpartner (axion) of inflaton. As is well known, *non-Gaussianity* is a measure of inflaton *interactions* described by its 3-point functions and higher – cf. eq. (24). The non-Gaussianity parameter $f_{\rm NL}$ is defined in terms of the (gauge-invariant) comoving curvature perturbations as

$$\hat{\mathcal{R}} = \hat{\mathcal{R}}_{\rm gr} + \frac{3}{5} f_{\rm NL} \hat{\mathcal{R}}^2_{\rm gr} \tag{212}$$

The non-Gaussianity was not observed yet, though it is expected. As regards the single-field inflationary models, they predict [94]

$$f_{\rm NL} = \frac{5}{12}(1 - n_s) \approx 0.02 \tag{213}$$

Finally, we would like to comment on possible testing of $f(R)$ gravity and $F(\mathcal{R})$ supergravity in Solar system and ground-based experiments.

As regards the large-scale structure of the present universe, the scalaron (ie. the dynamical spin-0 part of metric) may be responsible for its acceleration or Dark Energy (Sec. 11). However, since scalaron is universally coupled to all matter with gravitational strength, it may easily lead to a large violation of the equivalence principle. To avoid that, the scalaron should be essentially massless on the Solar system scales, because of the strong observational constraints from experimental tests of the equivalence principle [95, 96]. Moreover, it also should not give rise to a large violation of the equivalence principle in ground-based (on Earth) laboratories, because of the tight constraints on the fifth fundamental force in Nature [97].

A natural solution to both problems is provided by *Chameleon Cosmology* [98, 99], where the effective scalaron mass is dependent upon a local density ρ (see also refs. [100, 101]). The scalar potential of the scalaron (Chameleon) field gets an extra term, which gives rise to an effective scalar potential of the form

$$V_{\rm eff}(\varphi) = V(\varphi) + \rho \exp\left(\beta\varphi/M_{\rm Pl}\right) \tag{214}$$

where the parameter β is of the order 1. The exponential factor here arises due to the universal coupling of the scalaron to other matter of density ρ — see eq. (205). Therefore, when the effective mass is suffuciently large in a dense environment, one can evade the known constraints on the equivalnce principle and the fifth force, since the resulting violations are exponentially suppressed.

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